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Effect of a Transverse D.C. Electric Fields on Electrostatic Ion-Cyclotron Wave Instability

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Abstract

The temporal evolution of the current-driven electrostatic ion-cyclotron (CDEIC) instability is investigated in presence of a transverse d.c. electric fields in a collisional magnetized plasma. The growth rate of the instability has the largest value at the mode frequency for the appropriate magnetic field (where the effect of a transverse d.c. electric fields is larger) when the electron drift velocity is less than the critical value for the CDEIC instability. The growth rate is also a sensitive function of electron collision frequency.

Keywords:
ion cyclotron waves, d.c. electric fields, electron collision frequency, growth rate

1. Introduction

The first experimental observations of electrostatic ion cyclotron oscillations in a laboratory plasma were driven by applying a positive potential to a small electrode immersed in a uniform magnetic field in a single-ended Q-machine [1]. Several measurements of these oscillations and related phenomena were made under almost the same configuration and the results were discussed on the basis of current-driven instability [2]. Hatakeyama *et al.* [3] made measurements on the electrostatic ion cyclotron oscillations in nonuniform magnetic field under the same experimental configuration as in ref. [1] except that the axial profile of the magnetic field. The oscillation frequency was observed around the ion cyclotron frequency. Later Ganguli *et al.* [4] have presented a new kinetic ion cyclotron instability driven by a nonuniform electric field perpendicular to the external magnetic field, which could be of importance to the situations referred to ref. [3]. Schrittwieser *et al.* [5] have observed electrostatic ion-cyclotron instability driven by a slow electron drift in a single-ended Q-machine. Hatakeyama *et al.* [6] have presented a new measurements and mechanism of the ion cyclotron oscillations driven by applying a positive potential to the small electrode immersed in a magnetized plasma. Ganguli and Palmadesso [7] have found that the inclusion of a transverse localized electric field in addition to the magnetic field aligned current lowers the threshold for excitation and changes the dispersion characteristics of the waves. The influence of transverse, localized, d.c. electric fields on the current-driven electrostatic ion-cyclotron instability has been investigated in a Q-machine by Koepke *et al.* [8]. In this case, the ion cyclotron fluctuations were observed in presence of a transverse, d.c. electric fields when the electron drift velocity was less than the critical value for the CDEIC instability. Recently Amatucci *et al.* [9] have reported laboratory observations of reactively driven plasma waves in the ion cyclotron frequency range associated with a localized, transverse electric field. More recently, Koepke *et al.* [10] have observed

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multiple eigenmodes of the inhomogeneous energy-density driven instability with large and comparable amplitude. In this paper we develop a local theory of current driven electrostatic ion-cyclotron instability in presence of a transverse d.c. electric fields in a collisional magnetized plasma. Koepke *et al.* [8] have not taken into account transverse d.c. electric fields in their theoretical analysis. We have studied this effect using fluid theory.

2. Instability Analysis

Consider a homogeneous collisional plasma of equilibrium density n_{op}^{o}, temperature $T(= T_e$ for electrons and $= T_i$ for ions), collisional frequency $\nu(= \nu_e$ for electrons and $= 0$ for ions) immersed in a static magnetic field $\vec{B}_s \parallel \hat{z}$. We introduce a transverse, d.c. electric fields E_o in the x-direction. This initiates a drift of magnitude $v_E = cE_o/B_s$ in the y-direction. Moreover, electrons have a drift (v_{de}) along the magnetic field direction (i.e., z-axis) and we look for the excitation of ion cyclotron waves in the y-z plane. The equilibrium is perturbed by an electrostatic perturbation

$$\phi = \phi_o e^{-i(\omega t - k_y y - k_z z)}. \tag{1}$$

The response of the plasma electrons to the perturbation is governed by the equation of motion which on linearlization yields the perturbed velocity

$$v_{1x} = -\frac{eE_{1y}}{m}\frac{\omega_c}{[(\bar{\omega}+i\nu_e)^2-\omega_c^2]} + \frac{T_e}{im}\frac{k_y\omega_c}{[(\bar{\omega}+i\nu)^2-\omega_c^2]}\frac{n_{1e}}{n_{op}^{o}}, \tag{2}$$

$$v_{1y} = -\frac{eE_{1y}}{im}\frac{(\bar{\omega}+i\nu_e)}{[(\bar{\omega}+i\nu_e)^2-\omega_c^2]} + \frac{T_e}{m}\frac{k_y(\bar{\omega}+i\nu_e)}{[(\bar{\omega}+i\nu)^2-\omega_c^2]}\frac{n_{1e}}{n_{op}^{o}}, \tag{3}$$

$$v_{1z} = -\frac{eE_{1z}}{im(\bar{\omega}+i\nu_e)} + \frac{T_e}{m}\frac{k_z}{(\bar{\omega}+i\nu_e)}\frac{n_{1e}}{n_{op}^{o}}, \tag{4}$$

where $\bar{\omega} = \omega - k_y v_E - k_z v_{de}$; $-e$, m and $\omega_c(= eB_s/mc)$ are the electron charge, mass and cyclotron frequency, respectively, and subscript 1 refers to perturbed quantities. Using Eqs. (2), (3) and (4) in the continuity equation, we obtain the perturbed electron density

$$n_{1e} = \frac{n_{op}^{o}}{\bar{\omega}}\Big[\frac{ek_\perp^2\phi(\bar{\omega}+i\nu)}{m[(\bar{\omega}+i\nu_e)^2-\omega_c^2]} + \frac{T_e}{m}\Big[\frac{k_\perp^2(\bar{\omega}+i\nu_e)}{m[(\bar{\omega}+i\nu_e)^2-\omega_c^2]}\frac{n_{1e}}{n_{op}^{o}} - \frac{ek_z^2\phi}{m(\bar{\omega}+i\nu_e)} + \frac{T_e}{m}\frac{k_z^2 n_{1e}}{m(\bar{\omega}+i\nu_e)\,n_{op}^{o}}\Big]. \tag{5}$$

The response of plasma ions can be obtained from Eq. (5) by replacing $-e$, m, c, T_e and v_{de} by e, m_i, ω_{ci}, T_i and zero, respectively. Also we put $\nu_i = 0$ in Eq. (5) for ions, we obtain

$$n_{1i} = n_{op}^{o}\Big[\frac{ek_\perp^2\phi}{m_i(\acute{\omega}^2-\omega_{ci}^2)} + \frac{T_i k_\perp^2 n_{1i}}{m_i(\acute{\omega}^2-\omega_{ci}^2)} + \frac{ek_z^2\phi}{m_i\acute{\omega}^2} + \frac{T_i}{m_i}\frac{k_z^2}{\acute{\omega}^2}\frac{n_{1e}}{n_{op}^{o}}\Big], \tag{6}$$

where $\acute{\omega} = \omega - k_y v_E$. For electron response Eq. (5) is modified as

$$n_{1e} = \frac{n_{op}^{o}}{\bar{\omega}}\Big[-\frac{ek_z^2\phi}{m(\bar{\omega}+i\nu)} + \frac{T_e}{m}\frac{k_z^2}{(\bar{\omega}+i\nu)}\frac{n_{1e}}{n_{op}^{o}}\Big], \tag{7}$$

or

$$\frac{n_{1e}}{n_{op}^{o}} = \frac{e\phi}{T_e\Big[1-\frac{i\nu_e\bar{\omega}}{k_z^2T_e/m_e}\Big]}. \tag{8}$$

Equation (6) can be rewritten as

$$\frac{n_{1i}}{n_{op}^{o}} = \frac{ek_\perp^2\phi}{m_i(\acute{\omega}^2-\omega_{ci}^2)\,T_i\Big[1-\frac{T_i}{m_i}\frac{k_\perp^2}{(\acute{\omega}^2-\omega_{ci}^2)}\Big]}. \tag{9}$$

we have ignored axial wave vector k_z in Eq. (9) because $k_\perp^2 \gg k_z^2$ in the present analysis. Using quasi-neutrality approximation $n_{1e}/n_{op}^{o} \simeq n_{1i}/n_{op}^{o}$, we obtain $\varepsilon_r(\omega, k) + i\varepsilon_i(\omega, k) = 0$, where

$$\varepsilon_r(\omega,k) = \frac{(T_e+T_i)k_\perp^2}{m_i} - (\acute{\omega}^2-\omega_{ci}^2), \tag{10}$$

$$\varepsilon_i(\omega,k) = -\nu_e\bar{\omega}\frac{mk_\perp^2}{m_i k_z^2}. \tag{11}$$

writing $\omega = \omega_r + i\gamma$ and assuming that the wave is either weakly damped or growing (i.e, $|\gamma| \ll \omega_r$), the real part of the frequency ω_r is given by

$$\varepsilon_r(\omega = \omega_r, k) = 0, \tag{12}$$

or

$$\omega_r = k_y v_E + \sqrt{\omega_{ci}^2 + k_\perp^2 c_s^2}, \quad (13)$$

where $c_s^2 = (T_e + T_i)/m_i$. The imaginary part of the frequency is given by

$$\gamma = -\frac{\varepsilon_i(\omega, k)}{\partial \varepsilon_r(\omega_r, k)/\partial \omega_r}$$
$$= -\frac{1}{2}\frac{v_e[1 - \frac{k_z v_{de}}{\omega_r}(1+R)]}{(1 - \frac{k_y v_E}{\omega_r})}\frac{mk_\perp^2}{m_i k_z^2}, \quad (14)$$

where $R = k_y v_E / k_z v_{de}$. If $E_o = 0$ (in absence of a transverse, d.c. electric fields), i.e., $R \to 0$ we recover expression for the growth rate of Amatucci [11].

3. Results and Discussions

In the calculations we have used plasma parameters for the experiment of Koepke *et al.* [8]. Using Eq. (14), we have plotted in Fig. 1 the growth rate (in sec^{-1}) of the ion cyclotron wave instability as a function of mode frequency ω_r (in rad./sec) for the following parameters: finite gyroradius parameter $s = 1.0$, temperatures $T_e \simeq T_i \simeq 0.2$ eV, ion sound speed $c_s = 9.55 \times 10^4$ cm/sec, $m/m_i \simeq 1.3 \times 10^{-5}$ (potassium), guide magnetic field $B_s = 0.1$ kG–4.0 kG, plasma density $n_{op}^o \simeq 1 \times 10^9$ cm^{-3}, electron collision frequency $v_e \simeq 2.24 \times 10^5$ sec^{-1}, electron drift velocity $v_{de} = 6 \times 10^7$ cm/sec, $k_z v_{de}/r = 0.75$ (i.e., the electron drift velocity v_{de} is less than the critical value ω_r/k_z for the CDEIC instability), drift due to transverse d.c. electric fields $v_E = E_o/B_s$, where $E_o = 0.35$ statevolt cm^{-1} and mode frequency $\omega_r = 5 \times 10^5$ rad./sec to 12×10^5 rad./sec. From these parameters the value of R turns out to be $\ll 1$. For $R \ll 1$, the case of large v_{de} and small v_E, the electrostatic ion cyclotron waves are CDEIC instability. The growth rate of the instability increases with the mode frequency and has the largest value at the mode frequency for the appropriate magnetic field (where the effect of a transverse d.c. electric fields is larger) when the electron drift velocity is less than the critical value for the CDEIC instability. This result is similar to the experimental observations of Koepke *et al.* [8], where the ion cyclotron waves were observed in presence of a transverse, d.c. electric fields when the electron drift velocity was less than the critical value for the CDEIC instability. The growth rate of the instability is also a sensitive function of electron collision frequency.

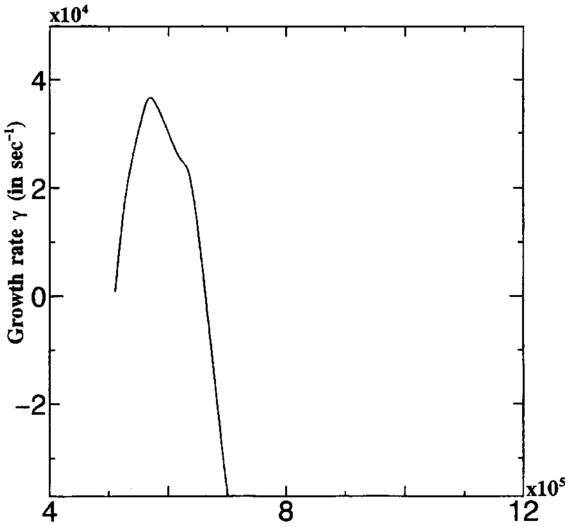

Fig. 1 Growth rate of the ion cyclotron wave instability (in sec^{-1}) as a function of mode frequency ω_r(in rad./sec). The parameters are given in the text.

In conclusion, we may say that electrostatic ion cyclotron waves are driven to instability by a low electron current in presence of a transverse d.c. electric fields. In the absence of transverse, d.c. electric fields the growth rate has the same value as obtained by Amatucci [11]. The growth rate of the instability increases with the mode frequency and has the largest value at the mode frequency for the appropriate magnetic field (where the effect of a transverse d.c. electric fields is larger) when the electron drift velocity is less than the critical value for the CDEIC instability in compliance with the experimental observations of Koepke *et al.* [8]. The frequency of the ion cyclotron waves increases with the magnetic field [cf. Eq. (13)].
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