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Electron Trapping and Acceleration in an Oblique Magnetosonic Shock Wave

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Abstract

Oblique magnetosonic shock waves are studied by means of a fully relativistic, fully electromagnetic particle code with full ion and electron dynamics, with particular attention to electron motion. It is found that some electrons can be trapped and then accelerated to ultra-relativistic energies in a shock wave; Lorentz factors with $\gamma \gtrsim 100$ have been observed in our simulations. The electron energies can be extremely high at certain propagation angles and shock speeds.

Keywords:
particle acceleration, ultra-relativistic electron, oblique shock wave

1. Introduction and Overview of Oblique Shock Waves

Ion acceleration by a shock wave has been shown and studied by a number of particle simulations [1-5] (and references therein). Strong electron acceleration in particle simulations, however, has not been reported, even though high-energy electrons have been observed in laboratory and astrophysical plasmas [6-8]. In this paper we will demonstrate by using a relativistic, electromagnetic particle simulation code that a large-amplitude, oblique shock wave can produce ultra-relativistic electrons.

We consider a magnetosonic wave propagating in the x direction with speed v_{sh} in an external magnetic field in the (x, z) plane. We assume that the field quantities depend on x only; $\partial/\partial y = \partial/\partial z = 0$. Then, the x component of the magnetic field is constant, $B_x = B_{x0}$. In the pulse region, B_y as well as B_z can have finite values.

In the wave frame where the time derivatives are zero ($\partial/\partial t = 0$), the electric field in the y direction is constant, $E_y = E_{y0}$, and E_z is zero. Hence we have $\boldsymbol{B} = (B_{x0}, B_y, B_z)$ and $\boldsymbol{E} = (E_x, E_{y0}, 0)$. The electric field E_{y0} is related to the shock speed v_{sh} through

$$E_{y0} = -v_{sh}B_{z0}/c. \tag{1}$$

We assume that $v_{sh} > 0$; and thus $E_{y0} < 0$.

The nonlinear wave theory tells us the wave structure. A shock wave will have a positive electric potential $\varphi(x)$. The quantities φ, B_z, and plasma density have similar profile [9,10]. On the other hand, E_x and B_y (and thus the electric field along the magnetic field $E_{\|}$) are proportional to the x derivatives of these quantities; for instance, $E_x = -\partial\varphi/\partial x$.

2. Physical Picture

We outline the mechanism why electrons can gain great energies if reflection occurs. Figure 1 shows a schematic diagram of the trajectory of electron guiding center in the wave frame. Here it is assumed that the reflection takes place at point D; the dotted line shows the orbit of a passing electron. As an electron moves from point A to C, it moves to the negative y direction because of the $E_x \times B_z$ drift. It gains kinetic energy ΔE_1

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from the electric potential; $\Delta E_1 = e\varphi(x_C) - e\varphi(x_A)$ (> 0). At the same time, it loses energy ΔE_2 because of the electric field E_{y0}; $\Delta E_2 = -eE_{y0}(y_C - y_A)$ (< 0). The net change in the energy is therefore

$$\Delta E = \Delta E_1 + \Delta E_2. \tag{2}$$

Even though the magnitudes of ΔE_1 and ΔE_2 are quite large, they almost cancel when an electron moves with drift approximation; in particular, in a perpendicular pulse they have exactly the same magnitude and $\Delta E = 0$ [11]. However, if an electron is reflected and moves from D to E, then it would gain energies from both E_x and E_{y0}. As a result, the increase in energy is

$$\Delta E = \Delta E_1 + \Delta E_3, \tag{3}$$

where ΔE_3 is defined as $\Delta E_3 = -eE_{y0}(y_E - y_A)$ (> 0).

3. Estimate of Energy Increase

We now obtain maximum energy of a reflected electron. The equation of motion for a relativistic electron is

$$m_e \frac{d(\gamma \boldsymbol{v})}{dt} = -e\left(\boldsymbol{E} + \frac{\boldsymbol{v} \times \boldsymbol{B}}{c}\right), \tag{4}$$

where γ is the Lorentz factor. For the stationary, one-dimensional system being considered, this can be integrated to give the energy conservation equation

$$m_e\gamma c^2 - m_e\gamma_0 c^2 = e(\varphi - \varphi_0) - eE_{y0}\int v_y dt. \tag{5}$$

We define quantity F as

$$F = -\int E_{\parallel}(B/B_{x0})\,dx. \tag{6}$$

It is noted that F has a profile similar to the quantities φ and B_z. Using F, we can rewrite Eq. (5) as

$$m_e\gamma c^2 - m_e\gamma_0 c^2 = e(F - F_0) - m_e c(E_{y0}/B_{x0})(\gamma v_z - \gamma_0 v_{z0}). \tag{7}$$

If we define quantity h as

$$h = m_e c^2 - m_e v_{sh} v_z B_{z0}/B_{x0}, \tag{8}$$

then from Eq. (7) we have

$$\gamma = [e(F - F_0) + h_0\gamma_0]/h. \tag{9}$$

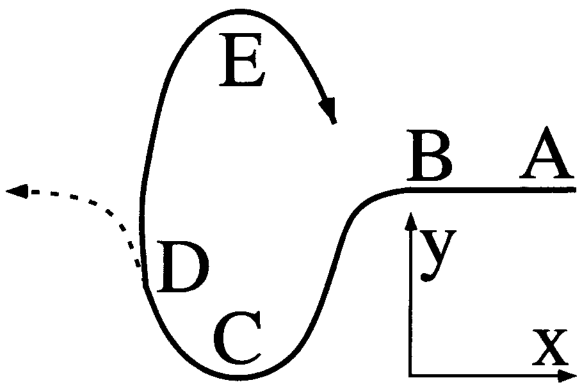

Fig. 1 Schematic diagram of electron orbit in the (x, y) plane.

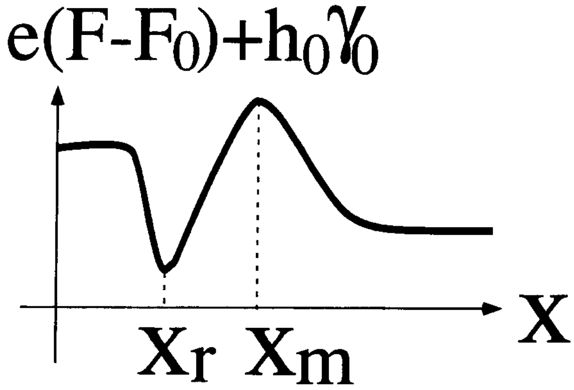

Fig. 2 Schematic diagram of quantity $[e(F - F_0) + h_0\gamma_0]$.

Equation (8) indicates that h is positive if B_{z0}/B_{x0} is of order unity. If B_{z0}/B_{x0} is much greater than unity, then h can be negative. In addition, Eq. (9) shows that, if the initial value of h is positive, $h_0 > 0$, then h is always positive in the region where $(F - F_0)$ is positive.

We show in Fig. 2 a schematic diagram of the function $[e(F - F_0) + h_0\gamma_0]$. We suppose that F has a maximum value at $x = x_m$ and has a minimum value right behind the main pulse. If the reflection occurs, it would be in this dip; x_r represents the reflection point. This function could be negative in the dip. We here, however, consider the case where it is always positive, because it can be shown that the strong acceleration takes place in this case.

We assume that an electron with positive h_0 is reflected at $x = x_r$. After the reflection, parallel electron velocity $v_{\parallel}$ increases and takes its maximum value at $x = x_m$, where φ and F are maximum. Then, Eq. (9) indicates that h has its minimum (positive) value at this point; hence, γ has its maximum value there.

We can estimate the maximum value γ_m in the wave frame by substituting the maximum value of F in Eq. (9). The maximum γ in the laboratory frame can be obtained by the Lorentz transformation as

$$\gamma_{lm} = \gamma_{sh} (1 + v_x v_{sh}/c^2)\, \gamma_m \simeq \gamma_{sh}\gamma_m. \quad (10)$$

Now let us consider a case where h becomes quite small, $h \simeq 0$, at $x = x_m$. There the electron speed would be close to the speed of light c. Hence, roughly, we have $v_z \sim cB_z/B$. The quantity h can then be expressed as

$$h = -m_e c^2 \left(1 - \gamma_{sh} \frac{v_{sh}}{c} \frac{B_z}{B} \tan\theta \right), \quad (11)$$

where $\tan\theta = B_{1z0}/B_{1x0}$. Here we have used the relations $B_{z0} = \gamma_{sh} B_{1z0}$ and $B_{x0} = B_{1x0}$. Because B_z becomes quite large in the shock region, we may put as $B_z/B \sim 1$. Equation (11) then suggests that the electron energy can become extremely large when

$$\gamma_{sh} (v_{sh}/c) \tan\theta = 1. \quad (12)$$

4. Simulation

We use a one-dimensional (one space coordinate and three velocity components), relativistic, electromagnetic, particle simulation code with full ion and electron dynamics to study the electron acceleration. The total system length is $L_x = 4096\Delta_g$, where Δ_g is the grid spacing. More detailed description about the simulation code can be found in Ref. [2].

The simulation parameters are as follows. The number of simulation particles is $N_i = N_e = 262{,}144$. The ion-to-electron mass ratio is $m_i/m_e = 100$. The ratio of the cyclotron to plasma frequencies is $\omega_{ce}/\omega_{pe} = 3.0$ in the far upstream region. The electron and ion thermal velocities are $v_{Te}/c = 0.38$ and $v_{Ti}/c = 0.01$, respectively. The Alfvén speed is $v_A/c = 0.3$. The time step is sufficiently small, $\omega_{pe}\Delta t = 0.02$, so that Δt is much smaller than the plasma and cyclotron periods even in the shock region. The external magnetic field is in the (x, z) plane, and waves propagate in the x direction. The propagation angle is $\theta = 45°$; i.e., $B_{x0} = B_{z0}$.

Figure 3 shows phase space plots of electrons in a shock with propagation speed $v_{sh} = 2.3\ v_A$. Some electrons are reflected near the end of the main pulse, and ultra-relativistic electrons are found in the shock region. The maximum value of their Lorentz factors is $\gamma \simeq 130$. We also see that p_x and p_z have greater values

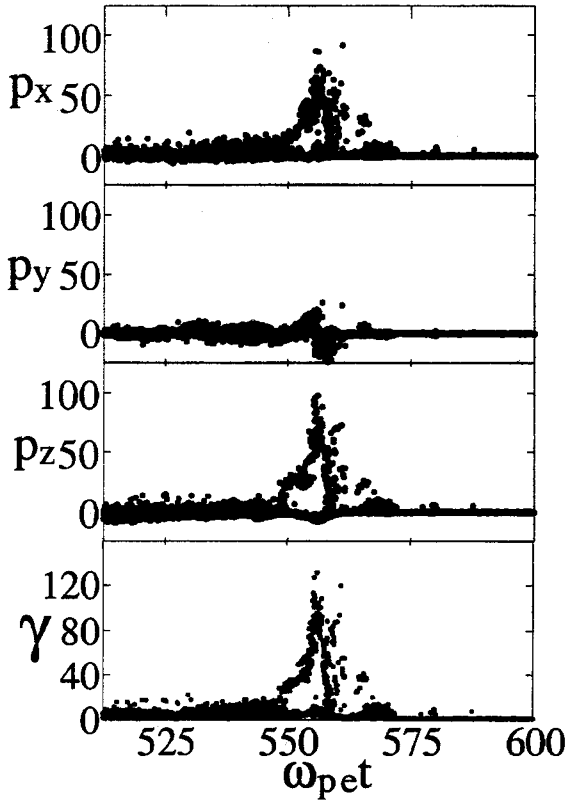

Fig. 3 Phase space plots of electrons.

than p_y; it is because the parallel velocity is dominant in the motion of high-energy electrons.

Next, we study trajectories of electrons; we will confirm that it is the reflected electrons that gain great energies. Figure 4 displays time variations of $(x - v_{sh}t)$, y, and γ of electrons in a shock with $v_{sh} = 1.75\ v_A$; $(x - v_{sh}t)$ is the x position in the wave frame. The lengths are normalized to the electron skin depth c/ω_{pe}. (Here the system size is a half of that in the previous run; the other parameters such as N_e/L_x are the same.) The thick lines show an electron that was accelerated, and the thin lines represent an electron that was not accelerated. In all of the three panels, the low- and high-energy electrons both have small-amplitude, cyclotron oscillations with short periods, $\omega_{pe}t \lesssim 20$. At the same time, the accelerated electron has a large-amplitude, long-period oscillation with period $\omega_{pe}t \simeq 170$; it begins when the particle enters the shock, $\omega_{pe}t = 180$, and continues until the end of this simulation run. For comparison with Fig. 1, we show the times corresponding to points B, C, D, and E. The top panel indicates that some electrons are reflected and then become high energy in the shock wave. On the other hand, the quantity ($x - v_{sh}t$) of the low-energy electron keeps decreasing, which means that this particle passes through the shock region without strong interactions. The second panel shows that the y position of the accelerated electron also oscillates.

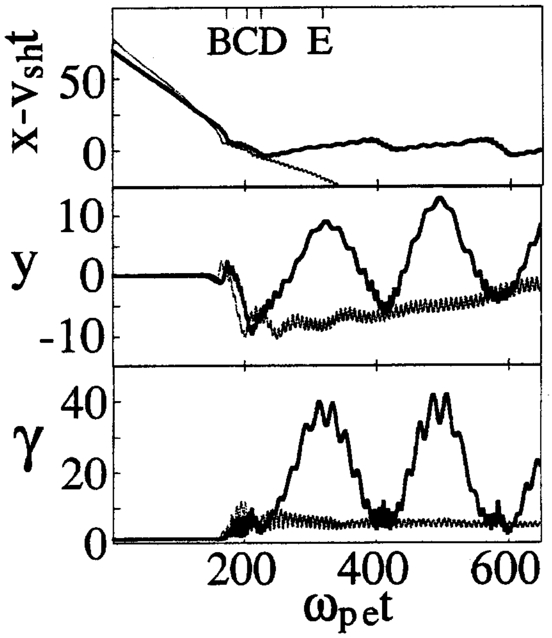

Fig. 4 Time variations of ($x - v_{sh}t$), y, and γ of passing and trapped electrons.

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