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Analysis of Current Diffusive Ballooning Mode in Tokamaks

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Abstract

The effect of finite gyroradius on the current diffusive ballooning mode is examined. Starting from the reduced MHD equations including turbulent transports, coupling with drift motion and finite gyroradius effect of ions, we derive a ballooning mode equation with complex transport coefficients. The eigenfrequency, saturation level and thermal diffusivity are evaluated numerically from the marginal stability condition. Preliminary results of their parameter dependence is presented.

Keywords:

ballooning mode, current diffusivity, turbulent transport, finite gyroradius effect

1. Introduction

The self-sustained turbulence model [1] of the current diffusive ballooning mode (CDBM) is one of the most promising candidates to explain the particle and energy transport in tokamaks. This transport model has successfully described the L mode transport and improved confinement associated with the formation of the internal transport barriers [2]. Since the previous analysis [3] based on the reduced MHD equations [4] neglected the finite ion gyroradius effect, we could not separate the transport coefficients of ions and electrons. In this paper, we describe the formulation and the results of numerical analysis of the ballooning mode equation which includes the coupling with the drift motion and the finite gyro radius effect.

2. Ballooning Mode Equation

We start from a set of reduced MHD equations, i.e. the equation of perpendicular motion of ions, the extended Ohm's law, the energy equations of electrons and ions [5],

$$\left(\frac{\partial}{\partial t}+\mathrm{i}\omega_{*\mathrm{i}}-\mu_\mathrm{i}\nabla_\perp^2\right)\nabla_\perp^2\bar{\phi}= \nabla_\parallel \tilde{J}_\parallel+(\boldsymbol{b}\times\boldsymbol{\kappa})\cdot\nabla(\tilde{p}_\mathrm{i}+\tilde{p}_\mathrm{e}), \tag{1}$$

$$\frac{\partial}{\partial t}\tilde{A}-\nabla_\parallel\tilde{\phi}=-\frac{1}{\omega_{\mathrm{ci}}\tau_\mathrm{A}}\nabla_\parallel\tilde{p}_\mathrm{e}-\lambda\nabla_\perp^2\tilde{J}_\parallel, \tag{2}$$

$$\frac{\partial}{\partial t}\tilde{p}_\mathrm{e}-\chi_\mathrm{e}\nabla_\perp^2\tilde{p}_\mathrm{e}+\boldsymbol{b}\cdot\left(\nabla_\perp\tilde{\phi}\times\nabla_\perp p_{\mathrm{e}0}\right)=0, \tag{3}$$

$$\frac{\partial}{\partial t}\tilde{p}_\mathrm{i}-\chi_i\nabla_\perp^2\tilde{p}_\mathrm{i}+\boldsymbol{b}\cdot\left(\nabla_\perp\bar{\phi}\times\nabla_\perp p_{\mathrm{i}0}\right)=0, \tag{4}$$

$$\nabla_\perp^2\tilde{A}=\tilde{J}_\parallel, \tag{5}$$

where $\tau_\mathrm{A} = qR/v_\mathrm{A}$, μ_i the ion viscosity, χ_i and χ_e the ion and electron thermal diffusivities, λ the current diffusivity due to the electron viscosity μ_e and other notations are standard. The streaming function for ions $\bar{\phi}$ is reduced from the electrostatic potential $\tilde{\phi}$ by a factor

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of $\Lambda_0(b) = I_0(b)e^{-b}$ where I_0 is the modified Bessel function of the first kind and $b = k_\perp^2\rho_i^2$. Length and time are normalized by the minor radius r and the poloidal Alfvén time τ_A, respectively. Using the ballooning transformation [6], we obtain

$$\frac{d}{d\eta}\frac{f}{-i\omega+\lambda n^4q^4f^2}\frac{d}{d\eta}\left(1-\frac{-i\omega_{*e}}{-i\omega+\chi_e n^2q^2f}\right)\tilde{\phi}$$
$$+\left(\frac{\alpha_e}{-i\omega+\chi_e n^2q^2f}+\frac{\alpha_i\Lambda_0}{-i\omega+\chi_i n^2q^2f}\right)H(\eta)\tilde{\phi}$$
$$+(i\omega-i\omega_{*i}-\mu_i n^2q^2f)f\Lambda_0\tilde{\phi}=0, \quad (6)$$

where $\alpha = -q^2R(d\beta/dr)$ is the normalized pressure gradient, $s = (r/q)(dq/dr)$ the magnetic shear and η the ballooning coordinates. Taking account of the Shafranov shift of the magnetic surface, we take $f = 1 + (s\eta - \alpha\sin\eta)^2$ and $H(\eta) = \kappa + \cos\eta + (s\eta - \alpha\sin\eta)\sin\eta$ where $\kappa = (r/R)(1 - 1/q^2)$ is the average magnetic curvature. The first term in eq.(6) has a stabilizing effect due to the bending of the magnetic field and is reduced by the current diffusivity λ. The second term coming from the pressure gradient destabilizes the mode in the bad curvature region and is reduced by the thermal diffusivity. The last term represents the stabilizing effect of ω_{*i} and the ion viscosity.

The transport coefficients are determined by the relation for the nonlinear transfer rate γ, e.g. $\chi_e = \langle\tilde{\phi}^2\rangle/\gamma$ and $\gamma = -i\omega + k_\perp^2\chi_e$, where the bracket $\langle\,\rangle$ denotes the average over the wave number and the η dependence of $\langle\tilde{\phi}^2\rangle$ is neglected in the present analysis [1]. Since mode frequency has a real part, the transport coefficients are written as

$$\mu_e=\xi\lambda=\sqrt{\frac{\langle\tilde{\phi}^2\rangle}{n^2q^2}-\frac{1}{4}\left\{\frac{\omega(\xi+n^2q^2)}{n^4q^4f^2}\right\}^2}+\frac{i}{2}\frac{\omega(\xi+n^2q^2)}{n^4q^4f^2}, \quad (7)$$

$$\mu_i=\sqrt{\frac{\Lambda_0^2\langle\tilde{\phi}^2\rangle}{n^2q^2}-\frac{1}{4}\left(\frac{\omega-\omega_{*i}}{n^2q^2f}\right)^2}+\frac{i}{2}\frac{\omega-\omega_{*i}}{n^2q^2f}, \quad (8)$$

$$\chi_e=\sqrt{\frac{\langle\tilde{\phi}^2\rangle}{n^2q^2}-\frac{1}{4}\left(\frac{\omega}{n^2q^2f}\right)^2}+\frac{i}{2}\frac{\omega}{n^2q^2f}, \quad (9)$$

$$\chi_i=\sqrt{\frac{\Lambda_0^2\langle\tilde{\phi}^2\rangle}{n^2q^2}-\frac{1}{4}\left(\frac{\omega}{n^2q^2f}\right)^2}+\frac{i}{2}\frac{\omega}{n^2q^2f}, \quad (10)$$

where n is the toroidal mode number and $\xi = \omega_p^2r^2/c^2$. When $\langle\tilde{\phi}^2\rangle$ is smaller than critical values, we take that the square roots have negative imaginary values so that the turbulent transport coefficients vanish for $\langle\tilde{\phi}^2\rangle = 0$. A pure imaginary value for small $\langle\tilde{\phi}^2\rangle$ implies nonlinear frequency shift rather than diffusion. We should note that the electron thermal diffusivity χ_e is less than χ_i by a factor of Λ_0 for large $\langle\tilde{\phi}^2\rangle$. With the increase of the perpendicular wave number $k_\perp \sim nq$, the finite gyroradius effect reduces χ_i.

With the marginal stability condition (Imω = 0), we numerically solve the ballooning mode equation (6) to find the eigen modes and evaluate the real part of the

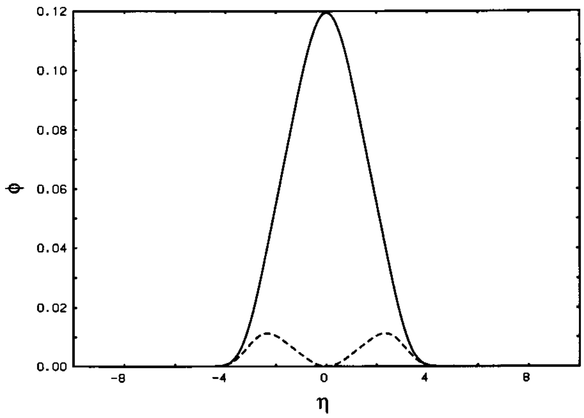

Fig. 1 Spatial structure of the eigen mode for the parameters s = 0.4, α = 0.4, q = 20, ρ/r = 0.01 and nq =134.

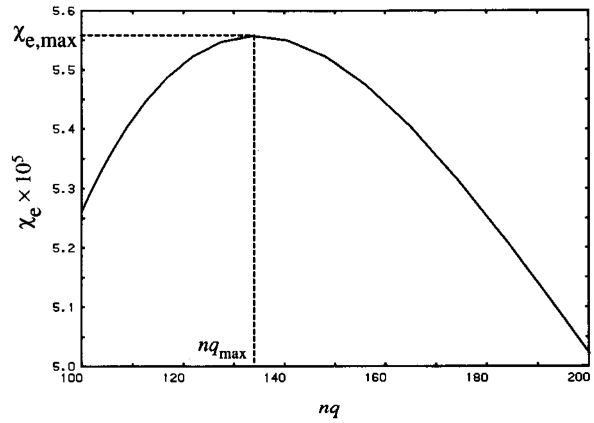

Fig. 2 nq dependence of the thermal diffusivity χ_e derived from the marginal stability condition for the parameters s = 0.4, α = 0.4, q = 20 and ρ/r = 0.01.

mode frequency ω_r and the average amplitude of the fluctuation $\langle\tilde{\phi}^2\rangle$. Fig. 1 shows an example of the CDBM eigenmode for $s = 0.4$, $\alpha = 0.4$, $q = 20$, $\rho_i/r = 0.01$ and $nq = 134$. It has a small negative real frequency ($\omega_r = -0.05$) and propagates in the direction of ion drift motion. The saturation level of the fluctuation is $\langle\tilde{\phi}^2\rangle = 6 \times 10^{-5}$ and this mode is unstable for smaller $\langle\tilde{\phi}^2\rangle$. The eigen mode structure indicates that the dominant real part localizes in the outer region ($\eta = 0$), while the out-of-phase imaginary part in the inner region ($\eta = \pm\pi$).

The saturation level of the average fluctuation amplitude depends on the mode number nq. Figure 2 illustrates the nq dependence of the electron thermal diffusivity χ_e derived from the saturation fluctuation amplitude $\langle\tilde{\phi}^2\rangle$. The maximum value of χ_e near $nq = 134$ gives the thermal diffusivity necessary for stabilizing the ballooning modes for all nq.

3. Parameter Dependence

In order to study the parameter dependence of the mode frequency and the thermal diffusivity, we look for the maximum value of χ_e by numerically solving the ballooning mode equation (6) for wide range of nq and for various values of s and α. We have been much benefitted by parallel processing on a PC cluster of 8 CPUs with a numerical code using the Message Passing Interface (MPI) library.

The (s, α) dependence of the electron thermal diffusivity is shown in Figs. 3 and 4 for $\xi = 10^4$ and $q = 10^3$. Fig. 3 depicts the case of small gyroradius, $\rho_i/r = 10^{-3}$, while Fig. 4 the case of larger gyroradius, $\rho_i/r = 10^{-2}$. The (s, α) dependence in Fig. 3 is almost same as that of $\rho_i/r = 0$ [2]. For larger ρ_i, χ_e is enhanced, typically, by a factor of 3 and the shear dependence is slightly reduced.

For small $\alpha(\alpha < 0.4)$, χ_e is a strongly increasing function of $\alpha(\chi_e \propto \alpha^{3/2})$ and decreases for small and negative values of s. For larger value of $\alpha(\alpha > 0.4)$, χ_e is enhanced in the stronger shear region ($s > 0.5$) where the ideal ballooning mode is unstable. The reduction of χ_e in the weak and negative shear region is more prominent for larger α (larger pressure gradient). These behavior is similar to the result of the previous analyses without coupling to the drift motion, $\omega_{*i} = \omega_{*e} = 0$. One of the reasons is the large q value in the present calculation ($q = 10^3$). For realistic values of q, the effect of finite ω_{*i} and ω_{*e} affects the mode structure, but conclusive results have not been obtained yet.

The mode frequency is usually negative except in the region, $\chi_e > 10^{-4}$, where the ideal mode is unstable.

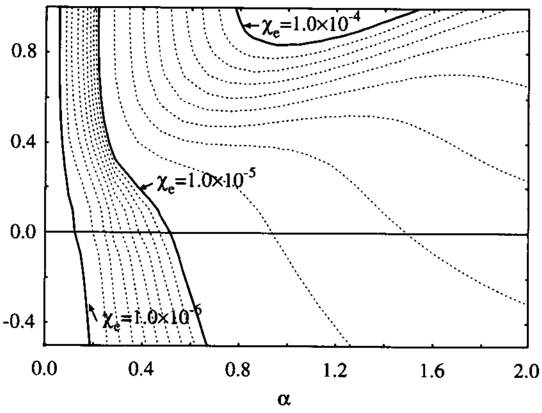

Fig. 3 Contour of the electron thermal diffusivity χ_e on the $(s-\alpha)$ plane for $\rho/r = 10^{-3}$.

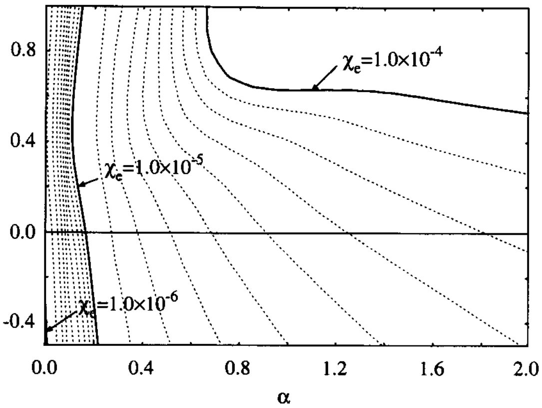

Fig. 4 Contour of the electron thermal diffusivity χ_e on the $(s-\alpha)$ plane for $\rho/r = 0.01$.

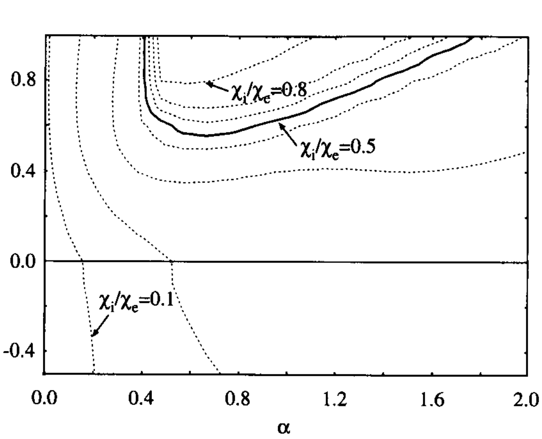

Fig. 5 Contour of the ratio χ_i/χ_e on the $(s-\alpha)$ plane for $\rho/r = 0.01$.

The mode number nq decreases with the increase of χ_e. Typical value of nq is 300 for $\chi_e = 10^{-5}$ and 100 for $\chi_e = 10^{-4}$. As the mode number increases, the finite gyroradius effect enhances χ_e and reduces χ_i. Fig. 5 indicates the contour of the ratio χ_i/χ_e on the (s, α) plane. In the region where $\alpha > 0.4$ and > 0.5, χ_i approaches to χ_e, otherwise χ_i/χ_e is less than about 0.4.

4. Conclusion

Taking account of the gyroradius effects, we found that χ_e of the CDBM transport model is enhanced by a factor of 3 for $\rho_i/r = 10^{-2}$ and that the ratio χ_i/χ_e is considerably reduced for low α(L-mode) region. In the present calculation, the safety factor q is limited to a rather large value. The calculation with realistic values of q as well as the effect of the $\boldsymbol{E} \times \boldsymbol{B}$ rotation shear will be reported in future.

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