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A Theory of Lead-Time in Probabilistic Excitation of L/H Transition

TODA Shinichiro, ITOH Sanae-I., YAGI Masatoshi,
ITOH Kimitaka[1] and FUKUYAMA Atsushi[2]
Research Institute for Applied Mechanics, Kyushu University, Kasuga 816-8580
(Present Address: National Institute for Fusion Science, Toki, Gifu, 509-5292)
[1]National Institute for Fusion Science, Toki, Gifu, 509-5292
[2]Department of Nuclear Engineering, Kyoto University, Kyoto, 606-8501 Japan

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Abstract

A quantity of a lead-time, t_{lead}, is newly introduced to examine the probabilistic occurrence of the L/H transition. The lead-time is a time period during which a transition is likely to occur. We show that the lead-time has the statistical distribution as a function of the distance from critical parameter, *e.g.* $|n_c - n_{c0}|$ when the density is a key parameter for transition. It has the dependence like $t_{\mathrm{lead}} \propto |n_c - n_{c0}|^2$ if the background noise distribution is given as $P(n_c) \propto |n_c - n_{c0}|^{-2}$.

Keywords:

probabilistic nature, L/H transition, lead-time, statistical variance, hysteresis

1. Introduction

Transition phenomena in plasmas (*e.g.*, L/H transition) are widely observed in various toroidal confinement devices. A transition (bifurcation) is modelled by the equation which includes a characteristic and cusp-type bifurcation. In the L/H transition study, the model equation is tested [1] to explain the experimentally-observed dithering ELMs [2]. This study clearly demonstrates that the high temperature plasma has a transition nature. Further observation of repetitive bifurcation was made on CHS device [3], and the importance of the study of transition nature is more widely recognized. Dynamics of transition in confined plasmas has been discussed by use of the critical condition [4]. However, to understand the nature of sudden onsets of L/H transitions, consideration of the basic physics mechanism is still necessary. There exist statistical variances in relevant variables. We have recently explored a new aspect of transition, that is, a probabilistic nature of occurrence. The cases in the presence of the Gaussian noise [5] and the noise with a power law have been examined [6]. It has been found that the transition occurs with a finite probability around the original threshold condition if the noise is present. The probability distribution around the threshold has been obtained. Here, we introduce another concept, which comes from the probability distribution, i.e., the lead-time. This statistical quantity is studied by the 0-D dynamical equation of transition, with noise effect, in which a hysteresis exists in the model of the loss rate.

2. Zero-Dimensional Dynamical Model Equations

We here adopt two basic equations: One is the temporal evolution of the plasma parameter. The other is the dynamics of the loss rate that produces the hysteresis of the flux-gradient relation. We choose two

Corresponding author's e-mail: toda@ms.nifs.ac.jp

representative variables, *i.e.*, the density and the loss rate γ in a layer with a finite width. The loss rate is directly related to the turbulence level and the particle diffusivity. The reduction to the 0-D model from the transport equation has been discussed [7]. Though highly simplified, the model form of the hysteresis has successfully applied to the investigation of the dynamics of dithering ELMs (which cycles between L-mode and H-mode). The model equation used here takes the forms

$$\frac{\partial}{\partial t} n = S - \gamma n \,, \tag{1}$$

and

$$\zeta \frac{\partial}{\partial t} \gamma = n - 1 + a(\gamma - 1) - b(\gamma - 1)^3 \,, \tag{2}$$

where S is the particle influx into the layer, $\zeta (= O(B_p^2/B_t^2))$ denotes the dynamical time difference between n and γ, and the cubic equation $a(\gamma - 1) - b(\gamma - 1)^3$ describes the shape of the hysteresis. Appropriate normalization is used for n, γ, and t.

The dynamical nature of the set of equations has been studied. If all the coefficients (S, a, b, ζ) are constant in time without noise, eqs. (1) and (2) predict the stable stationary solutions or the dynamical solution of a limit cycle. Stationary solutions are obtained for $S < S_1$ (lower flux branch, H-mode), $S_2 < S$ (higher flux branch, L-mode) and a limit cycle appears for $S_1 < S < S_2$, where $S_1 = (1 - \sqrt{a/3b})\,\{1 + (2a/3)\sqrt{a/3b}\}$ and $S_2 = (1 + \sqrt{a/3b})\,\{1 - (2a/3)\sqrt{a/3b}\}$. The transition from H-mode to L-mode takes place at $n_{c0} = 1 + 2a\sqrt{a/(3b)/3}$, *i.e.*, n_{c0} is the threshold condition in the absence of noise. We are here concerned this case as an example.

As the physics origin of the transport, the turbulence level is often discussed in terms of the statistical averages. The turbulence level is also associated with the statistical variance. For the confined plasmas, which is generally far from thermal equilibrium, the statistical variance is as important a quantity as the statistical average. The nonlinear simulation has shown a large temporal variation around the average [8]. The experimental observation has demonstrated that the statistical deviation from the mean value could be as large as the average itself [9]. Based on these considerations, we have been considered that parameters (S, a, b) are statistical variables and have fluctuation parts in time. In previous study [5,6], it has been found that the variance in has a stronger effect on the transition than that in S. Therefore, we here mainly focus the effect of the statistical variances of a. We set $a = a_0 + \varepsilon_a$ and consider ε_s and ε_a as statistical variables, *e.g.*, $\langle \varepsilon_a \rangle = 0$ and $\langle \varepsilon_a^2 \rangle \neq 0$. The variance ε_a comes from the deviation from the mean fluctuation level and causes the variation in the hysteresis characteristic.

3. Probabilistic Excitation of L/H Transition

We study the probabilistic nature which is caused by the statistical property of the turbulence. Temporal evolution are investigated by solving eqs. (1) and (2). The variation in a hysteresis characteristic is examined. The variance of ε_a is taken account with $\overline{\varepsilon_a} = \sqrt{\langle a - \langle a \rangle \rangle^2}$. The parameters are chosen as $a_0 = 0.5$, $b = 1.0$ and $\zeta = 0.01$. Accordingly, $S_1 \simeq 0.67$, $S_2 \simeq 1.21$ and $n_{c0} \simeq 1.136$. We set $S = 1.0$, *i.e.*, $S_1 < S < S_2$. We here introduce the noise, the probabilistic function for ε_a which obeys the power law, *i.e.*, $P(\varepsilon_a) \propto |\varepsilon_a|^{-2}$ in a certain region. The index of 2 is chosen as an example, but this number is not far from the observation of nonlinear simulation. In this example, the noise generator is used to obtain the fluctuation quantity ε_a in the domain $\varepsilon_a \geq 1 \times 10^{-6}$ and in $\varepsilon_a \leq -1 \times 10^{-6}$. In the region $1.2 \times 10^{-4} \leq |\varepsilon_a| \leq 2.5 \times 10^{-3}$, the relations $P(\varepsilon_a) \propto |\varepsilon_a|^{-2}$ and $P(\varepsilon_a) = P(-\varepsilon_a)$ hold. We set $P(\varepsilon_a) = P(1 \times 10^{-6})$ (= const.) for $|\varepsilon_a| \leq 1 \times 10^{-6}$. The variance is $\overline{\varepsilon_a} = 0.05$.

Oscillations like a limit cycle with irregular bursts are obtained. To show the probabilistic excitation of the plasma transition, the statistical distribution of the critical density n_c, at which the transition from H-mode to L-mode takes place, is observed. The distribution function of n_c, $P(n_c)$, is obtained in Fig. 1. This probability distribution clearly shows that the onset has the statistical nature. The value of n_c which takes the maximum probability nearly equals to n_{c0}, where the transition occurs in the absence of noise. In this statistical fluctuations, we observe $\delta n/\delta a \simeq 0.37$, where

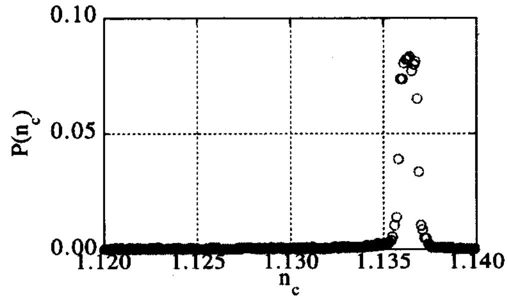

Fig. 1 The probability distribution for the density at the onset of the transition. Here, $S = 1.0$, $a_0 = 0,5$, $b = 1.0$ and $\zeta = 0.01$.

$\delta n = n_{c,max} - n_{c,min}$ and $\delta a = a_{c,max} - a_{c,min}$. The dependence of $\log_e P(n_c)$ on $\log_e|n_c - n_{c0}|$ is examined in Fig. 2. The solid line indicates the relation $P(n_c) \propto |n_c - n_{c0}|^{-2}$. (A linear fitting in the region $1.2 \times 10^{-4} \leq n_{c0} - n_c \leq 1.2 \times 10^{-3}$ gives $\log_e P(n_c) = M_0 + M_1 \log_e|n_c - n_{c0}|$ with $M_0 = -19$ and $M_1 = -1.99$ and the correlation coefficient $R = 0.98$. Scattering in the tail is statistical noise and we deduce in such a form like $R = 1-145/I_{tot}$ ($1000 < I_{tot} < 10000$) when the number of observation of transition, I_{tot}, is increased. The distribution function is found to obey the same power law $P(n_c) \propto |n_c - n_{c0}|^{-2}$ as the generated noise. Analysis about other critical parameters such as period and particle flux have been done for the different type of noise [6]. The similar result is also obtained in this study.

4. Analysis about the Lead-Time

The finite probability of the transition below the criterion $n_c < n_{c0}$ allows one to estimate the effective lead-time before the transition occurs. As in the previous section, we here study the transition from the H-mode to L-mode. A time sequence of repeated transitions (total number of I_{tot}) in dithers is considered as a set of I_{tot} observations of the transition events. In each temporal evolution of transition, the initial condition is chosen as the H-state, and the onset of H- to L-transition is observed. The ratio that the plasma state remains the H-mode (the transition does not occur yet) is set N as the mean value for the case of I_{tot} times of the repeated transitions. Then the rate equation is given as $-dN/dt = P(n(t))/\tau_{tr}$ ($= P_{tr} N$), where τ_{tr} is the time scale to complete one transition event after being triggered and P_{tr} is the transition rate. The transition takes place with an expected rate $\tau_{tr}^{-1}P(n(t))$. In this article, we choose the normalized value $\tau_{tr} = 1$ without lack of generality. So the model equation is given as

$$-\frac{\mathrm{d}N}{\mathrm{d}t} = P(n(t)). \tag{3}$$

The partition rate $N(t)$ is calculated as

$$N(t) = 1 - \int_0^t P(n(t'))\,\mathrm{d}t', \tag{4}$$

where we assume that the value of $n(t = 0)$ is enough far from n_{c0} and the transition never occurs at $t = 0$, *i.e.*, $N(t = 0) = 1$. Without the noise, the model equation is simplified as $-dN/dt = \delta(t - t_*)$, where $\delta(t - t_*)$ is the Dirac's delta function and $n(t = t_*) = n_{c0}$. In this case, the transition always occurs at $n = n_{c0}$. The rate $N(t)$ is evaluated in the equation as $N(t) = 1 - \int_0^t \delta(t' - t_*)dt'$. The value of $N(t)$ in the region $t < t_*$ tis unity and the one in the region $t > t_*$ is zero. The ratio N has the discontinuity nature and $|dN/dt|_{t=t_*} = \infty$. In the presence of the noises, the states $N(t = 0)$ and $N(t = \infty)$ are found to be smoothly connected. This fact indicates that the parameter N is reflected by the probabilistic nature in the L/H transition. The quantity $N/|dN/dt|$ is an estimate of the effective time against the transition phenomena and the change time for the state. We here call this time 'lead-time': t_{lead} as shown in Fig. 3. Without noises, t_{lead} is infinity and no transition takes place in the region $t < t_*$. At $t = t_*$, t_{lead} is zero and the transition always happens. However, if the noise is present, the transition happens at $n < n_{c0}$ with a finite probability of $\int_0^{t_{lead}} P(n)dt$ after the time interval t_{lead}. In this case investigated here, the lead-time is shown to have the dependence such as $t_{lead} \propto |n_c - n_{c0}|^2$ which has inverse dependence for the background noise in an analytic insight. The dependence of $\log_e(N/|dN/dt|)$ on $\log_e|n_c - n_{c0}|$ is examined in Fig. 4. The solid line indicates the relation $(N/|dN/dt|) \propto |n_c - n_{c0}|^2$. (A linear fitting in the region $5.5 \times 10^{-4} \leq n_{c0} - n_c \leq 2.5 \times 10^{-3}$

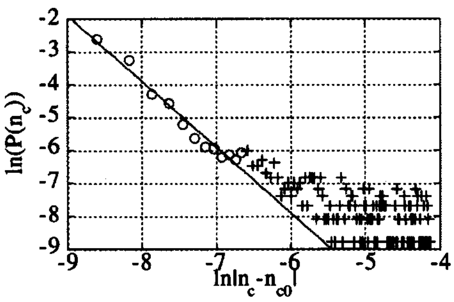

Fig. 2 The dependence of $\log_e P(n_c)$ on $\log_e|n_c - n_{c0}|$ is shown. The value of the slope for the solid line is −2.

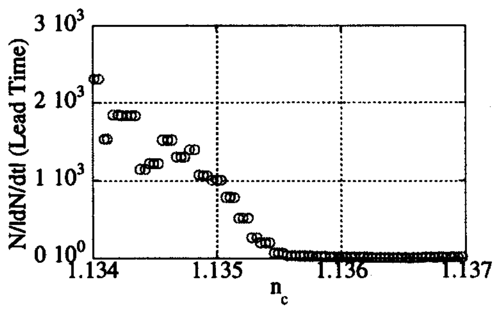

Fig. 3 Lead Time ($N/|dN/dt|$) is shown as the function of n_c.

gives $\log_e(N/|dN/dt|) = M_2 + M_3 \log_e|n - n_{c0}|$ with $M_2 = 20$ and $M_3 = 2.01$ and the correlation coefficient $R = 0.94$. It is found that the numerical result shows the dependence $t_{\rm lead} \propto |n_c - n_{c0}|^2$ similar to the analytic one. This fact is confirmed in the cases for other values of S.

5. Summary and Discussions

In this article, we study the probabilistic nature of the transition by 0-D model equation. The statistical variance for relevant parameters expresses the probability of the transition based on the probabilistic view. The distribution of critical parameters is confirmed to reflect the statistical property of the background turbulent field for the case $P(\varepsilon_a) \propto |\varepsilon_a|^{-2}$. Next a concept of lead-time is naturally introduced to extend the probabilistic picture for the transition phenomena. This time is the average time period, during which a transition is likely to occur. The lead-time is found to have the dependence of the inverse of that of the background noise field. The statistical study on the experiments of transition will provide a unique information to understand the physics of the transition as well as the nature of the turbulence in plasmas. Note that we show the case for the one kind of noise in this paper. Cases for other noises are left for future work.

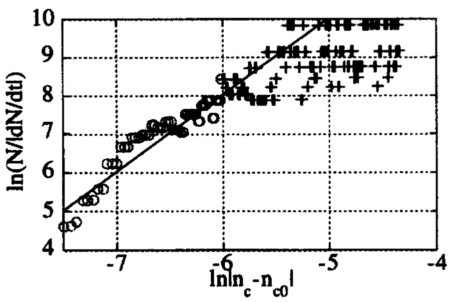

Fig. 4 The dependence of $\log_e(N/|dN/dt|)$ on $\log_e|n_c - n_{c0}|$. The value of the slope for the solid line is 2.

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