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Interesting Differences between Axisymmetric and Non-Axisymmetric Systems

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Abstract

Differences between axisymmetric and non-axisymmetric toroidal systems are reviewed from the various viewpoints of MHD equilibrium, particle orbits, neoclassical transport, and ideal MHD spectrum and instability.

Keywords:
axisymmetric and non-axisymmetric toroidal system, MHD equilibrium, ideal MHD spectrum and instability, neoclassical transport

1. Introduction

Differences between axisymmetric and non-axisymmetric toroidal systems come from whether symmetry in the toroidal direction exists or not. In that existence of the ignorable coordinate reduces three-dimensional problems into two-dimensional ones, axisymmetric toroidal systems are regarded as physically degenerate states of non-axisymmetric toroidal systems. Thus, phenomena inherent to non-axisymmetric toroidal systems exist together with many common features to axisymmetric toroidal systems. In this paper, we will concentrate our attention on the differences between both systems, or phenomena inherent to non-axisymmetric toroidal systems.

2. MHD Equilibrium and Particle Orbits

In axisymmetric toroidal systems, MHD equilibria consist of clear nested flux surfaces and collisionless single particle orbits have the invariant of motion. In non-axisymmetric MHD equilibria, lack of the toroidal symmetry leads to breaking up the flux surfaces into magnetic islands, and to stochastic motion of particle orbits. Thus, a kind of optimization of the non-axisymmetric MHD equilibrium as was done in LHD and W7-X is needed in order to avoid them as much as possible. If such an optimization is not enough, then large size of magnetic islands and/or splitting of the magnetic axis occur as β-value increases, especially for low magnetic shear systems. Although existence of magnetic islands by themselves brings about profound accademic and practical problems, we could still find fruitful interesting differences between axisymmetric and non-axisymmetric toroidal equilibria under the assumption of clear nested flux surfaces. Under this assumption, differences appears as the situation whether equilibrium quantities depend on the toroidal angle or not, when we use an appropriate magnetic coordinate system such as the Boozer coordinates. In other word, equilibrium quantities depend on not only poloidal but also the toroidal Fourier modes in non-axsymmetric toroidal systems. Hereafter, we consider $L = 2$ planar axis heliotron configurations with good flux surfaces like LHD, where L is the polarity of helical coils.

3. Neoclassical Transport

The neoclassical transport is determined by particle orbits under the Coulomb collision, which is expressed in terms of so called flux-thermodynamic force relationship. Since the lowest order distribution function is the local Maxwellian, two kinds of the thermodynamic force appear for each particle species (if no external field exists):

$$A_{a1}(\psi) = \frac{1}{e_a n_a(\psi)} \frac{\mathrm{d}P_a(\psi)}{\mathrm{d}\psi} + \frac{\mathrm{d}\Phi(\psi)}{\mathrm{d}\psi},$$

$$A_{a2}(\psi) = \frac{1}{e_a} \frac{\mathrm{d}T_a(\psi)}{\mathrm{d}\psi},$$

where the standard notation is used, and Φ and ψ are the electrostatic potential and the label of the flux surface, respectively. The lowest order mean flows and heat fluxes are determined by the flux-surface averaged perpendicular and parallel force balances for each particle species. Both are incompressible ones on flux surfaces. It are these lowest order quantities that determine the same order bootstrap current and the next order radial particle and heat fluxes. Differences of both systems come from 1) existence or non-existence of the toroidal viscosity making a non-ambipolar radial flux, and 2) the form of the parallel viscosity. In axisymmetric toroidal systems with symmetry in the toroidal direction, the toroidal viscosity disappears, so that all the radial fluxes become intrinsically ambipolar. It means that the ambipolar condition does not determine the radial electric field $\mathrm{d}\Phi/\mathrm{d}\psi$. Toroidal symmetry ensures the conservation of the lowest order total toroidal angular momentum, and hence only the poloidal flow (flow in the direction without symmetry) is selectively damped by the parallel viscosity, independent of the collisionality regime of each particle species. The resultant poloidal flow has no component due to the radial electric field, and only the toroidal flow has such a component. The fact that an undetermined radial electric field appear only on the toroidal flow is consistent to the conservation of the total toroidal angular momentum, because such a conserved quantity should be determined from an appropreate initial condition, and hence an appropreate unknown quantity must be included there. Moreover, the bootstrap current does not include the component directly proportional to the radial electric field, inspite of the form of the thermodynamic force $A_{a1}(\psi)$. Therefore, undetermined radial electric field does not influence the MHD equilibria including the bootstrap current.

In non-axisymmetric toroidal systems, lack of the toroidal symmetry leads to a non-ambipolar diffusion due to the toroidal viscosity, which is inversely proportional to the Coulomb collision frequency ($1/\nu$-ripple diffusion). Thus, ambipolar condition determines the radial electric field. Due to lack of toroidal symmetry, the parallel viscosity damps not only poloidal flow, but also toroidal flow, and the direction of damping depends on the collisionality regime of each particle species. Thus, both poloidal and toroidal flows have a component proportional to the radial electric field, and the bootstrap current has a component directly proportional to the radial electric field, when electrons and ions exist in the different collisionality regimes. Consequently, the determinable radial electric field directly influences MHD equilibria through the bootstrap current in the non-axisymmetric toroidal systems. This influence becomes significant when electrons and ions exist in the $1/\nu$ and plateau collisionality regimes, respectively. In this situation, the radial electric field determined by the ambipolar condition reduces the bootstrap current [1,2].

As is well known, the diamagnetic current and Pfirsch-Schlüter current are independent of the radial electric field in both systems, because that the diamagnetic current is created by the local gyration motion around a magnetic field line, and that Pfirsch-Schlüter current flows so as to satisfy incompressibility of the current. In contrast, bootstrap current comes from the global drift motion of the guiding center, which can feel the global structure of the magnetic field configuration, namely, symmetry and asymmetry.

4. Ideal MHD Spectrum and Instability

4.1 Shear Alfvén continuum and spectral gaps

In one-dimensional systems such as straight tokamaks with circular cross section, equilibrium quantities describing shear Alfvén branch are surface quantities, which make shear Alfvén continuum in the stable side of the ideal MHD spectrum. In axisymmetric (two-dimensional) toroidal systems, in which equilibrium quantities have poloidal angle dependence, this shear Alfvén continuum has spectral gaps due to poloidal mode couplings between equilibrium quantities and perturbations. In non-axisymmetric (three-dimensional) toroidal systems, equilibrium quantities have both poloidal and toroidal angle dependences, and hence the spectral gaps in the shear Alfvén continuum are created by both poloidal and toroidal mode couplings. The behaviour of the shear Alfvén continuum is shown in Figs.1-(a) and (b), where an MHD equilibrium in an $L = 2/M = 8$ planar axis heliotron configuration is used (L and M are polarity and toroidal field period of helical coils, respectively). In Fig.1-(a), the shear Alfvén spectrum with both poloidal and toroidal mode couplings is shown where Fourier modes are used satisfying $n \pm n_1 = \bar{n}M$, $n_1 = 2$, $\bar{n} = 0, \pm1, \pm2,$

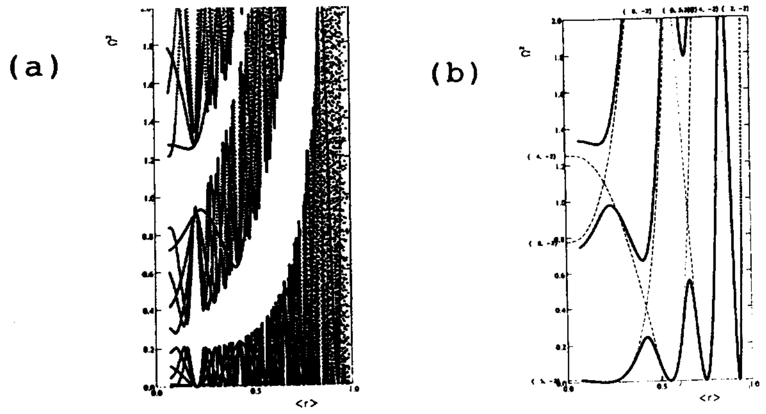

Fig. 1 Shear Alfvén spectrum (a) with poloidal and toroidal mode couplings, and (b) without mode coupling (dsahed lines) and with only poloidal mode coupling (stars).

$\cdots$, namely, $n = \cdots, -10, -6, -2, 2, 6, 10, \cdots$. From these figures, it is understood that the coexistence of poloidal and toroidal mode couplings dramatically changes the spectrum gap structure in the shear Alfvén continuum. Toroidicity-induced shear Alfvén Eigenmodes (TAE) are considered to change compared with those in axisymmetric toroidal systems.

4.2 Continuous unstable spectrum of pressure-driven modes

The coexistence of poloidal and toroidal mode couplings might lead to the possibility to make a continuous or quasi-point unstable spectrum. Interchange modes have the properties to localize around their mode rational surfaces. In one-dimensional equilibria, both poloidal m and toroidal n mode numbers are specified. For a particular selection (m, n), countable infinite number of eigenvalues can exist, which come from countable infinite number of radial mode (node) numbers. In axisymmetric (two-dimensional) equilibria, only a toroidal mode number is specified, and hence an overall structure of the interchange mode consists of the superposition of each Fourier mode localized around the mode rational surface, namely, $\iota = \cdots, n/(m-1), n/m, n/(m+1), \cdots$, where ι is the rotational transform. Note that since the toroidal mode number is same, each Fourier mode localizes at the different rational surface. Since the interchange modes are driven by the averaged unfavarable magnetic curvature, those amplitude are almost constant along their mode rational magnetic field lines, and each Fourier mode is basically independent. Therefore, the interchange mode constructed by the superposition of Fourier modes with different radial mode numbers and/or different relative phase can also become unstable with different eigenvalues. Thus, as well as one dimensional systems, interchange modes in axisymmetric equilibria will have countable infinite number of eigenvalues. In axisymmetric (two-dimensional) equilibria, each Fourier mode constructing interchange mode localizes at the different rational surface, and hence essential structure are same as one-dimensional one. In non-axisymmetric (three-dimensional) equilibria, the situation is different, because two modes with different combination of Fourier modes, namely, (m, n) and (m', n') may have the same mode rational surface due to the toroidal mode coupling, namely, $\iota = n/m = n'/m'$, where n' satisfies $n' \pm n = \bar{n}M$ for a particularly chosen integer $\bar{n}$. The superposition of Fourier modes with the same rational surface could make a continuous variation of the eigenfunction and the eigenvalue, leading to a continuous unstable spectrum, or quasi-point unstable spectrum.

In the case of ballooning modes, the situation is different from that of interchange modes. Ballooning modes are driven by the locally unfavorable magnetic curvature, and hence they have a tendency to be localized around there. On the other hand, the superposition of Fourier modes with different toroidal mode numbers, by changing their relative phase, leads to the change of the localization in the toroidal direction. Therefore, the change of the superposition of Fourier modes with different toroidal mode number is easy to lead to stabilization, and hence ballooning modes might not have a continuous unstable spectrum or a quasi-point unstable spectrum, execpt perhaps for the case with extremely high toroidal mode numbers. Above consideration is different from that in Ref. [3].

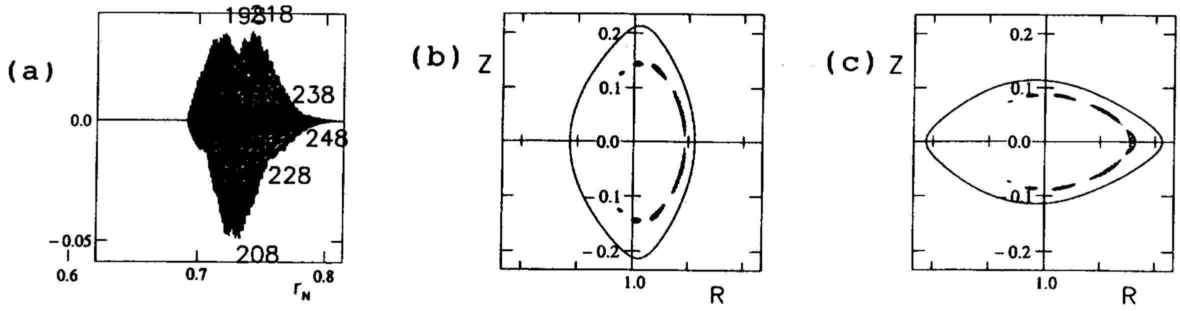

Fig. 2 Ballooning modes inherent to non-axisymmetric systems. (a) radial distribution of Fourier modes of $\vec{\xi}\cdot\nabla\psi$ with their dominant toroidal mode numbers, and corresponding contours of the perturbaed pressure $\tilde{P} = -\nabla P \cdot \vec{\xi}$ on the vertically (b) and horizontally (c) elongated poloidal cross section.

4.3 Pressure-driven (ballooning) instabilities

In the limit of a short perpendicular wave length (the local mode analysis), ballooning modes are analyzed by using the ballooning equation in the covering space (ψ, η, α), where ψ and α are the labels of a flux surface and a magnetic field line on it, respectively, and η is a coordinate along the magnetic field line.

In axisymmetric toroidal systems, the eigenfunction ξ and the eigenvalue ω^2 have functional dependence: $\xi = \xi(\eta|\psi, \theta_k)$, $\omega^2 = \omega^2(\psi, \theta_k)$, where θ_k is the radial wave number stemming from the eikonal approximation. Thus, level surfaces of unstable eigenvalues ω^2 (< 0) are topologically cylindrical with the axis in the α direction in (ψ, θ_k, α) space. For a specified toroidal mode number n, semiclassical quantization condition determines the eigenvalue of the global mode: $\omega^2 = \omega^2(n)$, and the ballooning representation can determine the global mode structure $\xi = \xi(\vec{r}, n)$. Thus, the local mode analysis leads to the global mode analysis.

In non-axisymmetric toroidal systems, the local magnetic shear having a stabilizing effects disappears due to a large Shafranov shift even in the region with the stellarator-like global magnetic shear, leading to the destabilization of ballooning modes there [4]. Because of the toroidal angle dependence of the MHD equilibria coming from helical coils, the magnetic curvature consists of two parts due to both toroidicity and helicity [5]. The eigenfunction and the eigenvalue have functional dependence: $\xi = \xi(\eta|\psi, \theta_k, \alpha)$, $\omega^2 = \omega^2(\psi, \theta_k, \alpha)$. The field line dependence α is due to the toroidal symmetry breaking, and is mainly caused by the helicity part of the local magnetic curvature [5]. Due to the field line dependence of the eigenvalue, there are two types of level surfaces of the unstable eigenvalue in (ψ, θ_k, α) space, namely, a tokamak-like topologically cylindrical level surface and a topologically spheroidal level surface. In Mercier-unstable MHD equilibria, both cylindrical and spheroidal level surfaces coexist, and in Mercier-stable MHD equilibria, only spheroidal level surfaces exist [5]. Since the local mode analysis does not lead to the global mode analysis due to α dependence of the eigenvalue, it is conjectured that the global modes corresponding to the eigenvalues with cylindrical level surfaces are tokamak-like ballooning modes or interchange modes, and that the global modes corresponding to the eigenvalues with spheroidal level surfaces are ballooning modes inherent to non-axisymmetric toroidal systems with localized in both the poloidal and toroidal directions [5]. Recently, the above conjecture has been confirmed by the global mode analysis. One example of the ballooning modes inherent to non-axisymmetric toroidal systems is shown in Fig.2. There are eight groups of Fourier modes for $\vec{\xi} \cdot \nabla\psi$ with different toroidal mode numbers through the strong toroidal mode coupling in Fig.2-(a). Each group has a similar structure to that of ballooning modes in tokamaks by the poloidal mode coupling. Due to these poloidal nad toroidal mode couplings, the perturbed pressure $\tilde{P} = -\nabla P \cdot \vec{\xi}$ localizes in both the poloidal and toroidal directions, leading to the localization on some flux tubes as shown in Figs.2-(b) and (c).

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