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Enhancement of Runaway Avalanche by Lower Hybrid Waves

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Abstract

The runaway avalanche process is different from the Dreicer runaway mechanism by the much smaller threshold electric field. The "knock-on" source produced by large angle Coulomb scattering is a key ingredient for secondary energetic electron generation which reduces the electric field requirement. The lower hybrid waves, by effectively accelerating the secondary electrons, can alter the growth rate and magnitude of the avalanche process. This paper presents the first demonstration by numerical simulation of the Fokker-Planck equation, including large angle Coulomb scattering, that significant enhancement of the runaway production rate is possible.

Keywords:
runaway electrons, lower hybrid waves

1. Introduction

The "knock-on" source is a key ingredient in recent studies of secondary electron production by runaway avalanche during tokamak disruptions [1] and disruption mitigation using giant pellet injection [2]. The avalanche depends on the existence of some trace amount of very energetic electrons (MeV and above) in the plasma. These electrons can knock out bulk electrons to supercritical energies (*i.e.*, above the energy where the electric field acceleration balances the collisional drag) through large angle collisions. They become runaways and can positively feedback on the knock out process resulting in an avalanche effect. This process is distinct from the process discussed by Dreicer [3] by the much smaller threshold electric field E_c (smaller than the Dreicer field E_D by v_t^2/c^2) and the relatively long growth time for the knock-out runaways to build up. A comprehensive, relativistic theory of knock-on avalanche has been developed, and has been validated by Monte-Carlo simulations [4] and Fokker-Planck calculations [5]. A momentum orbit analysis has also analytically reproduced the runaway avalanche growth rate [6].

The present study considers how the lower hybrid (LH) waves can affect the runaway avalanche process. Even when the minimum phase velocity $\omega/k_\parallel$ of the lower hybrid spectrum is significantly larger than the electron thermal velocity (hence is not expected to produce a large current according to quasilinear theory), the waves can still effectively generate an initial high energy tail with much higher density than any considered in the disruption scenario. One would then expect the growth of the run-away avalanche current to be much more expedient, and should increase to observable magnitude within the duration of present day tokamak operation. Furthermore, extending the orbit analysis to this case, the avalanche growth rate is shown to be also significantly enhanced by the LH waves. In the following, a model describing the characteristics of the runaway avalanche current, and the dependence of its amplitude and growth rate on wave and electric field parameters is presented. This model is compared against

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Fokker-Planck simulations which clearly demonstrate the existence of an enhancement effect.

This result suggests that runaway avalanche physics can be validated in existing tokamaks with limited volt-seconds through the use of LH waves. It also offers a new explanation for the LH "spectral gap" problem, an issue having to do with the discrepancy between the need of low $\omega/k_{\|}$ velocity to justify the prediction of quasilinear theory and the low $n_{\|} = k_{\|}c/\omega$ values of the spectra launched in some ex periments, which has not been resolved with complete satisfaction [7].

2. Enhanced Runaway Current by "Knock-On" Avalanche and LH Waves

The Landau-Fokker-Planck collision operator treats only the dom-inant small-angle Coulomb scattering contribution. To study "knock-on" effects one has to modify the Fokker-Plank equation by adding a source term S to account for the electrons knocked out from the bulk to large energies through large-angle scatterings. Furthermore, at least a weakly relativistic treatment (justified for $T/mc^2 \ll 1$) is required. The following assump-tions are used in the derivation of the source. In a two particle collision process, the slower particle from the bulk is taken to have zero initial momentum, while the high energy (runaway) particle has finite momentum only parallel to the electric field, *i.e.*, $\vec{w}_1' = w_1'$ initially. The high energy particle is scattered to momentum $\vec{w}_1$, knocking out the idle electron to momentum $\vec{w}$. The relationship of the initial and final momenta can be derived using conservation of momentum and energy. This relationship is geometrically expressed by the equation of an ellipse

$$\frac{w_\perp^2}{2(\gamma_1'-1)} + \left(\frac{w_\|}{\sqrt{\gamma_1'^2-1}} - \frac{1}{2}\right)^2 = \frac{1}{4}, \qquad (1)$$

where $\gamma^2 = 1 + w^2$, $w = u/c$. In the limit $\gamma_1' \to \infty$, Eq. (1) becomes parabolic: $w_\| = w_\perp^2/2$. Using the conservation of particles in the laboratory and CM frame, the source S is related to the Møller cross-section [8] which accounts for large-angle scattering. The complete form is given in Ref. [5]. It is useful for analytic evaluation of the growth rate to change variables from $(w_\|, w_\perp)$ to (w, μ) so in the limit $\gamma_1' \to \infty$

$$S = \frac{n_r \ln(\Lambda)}{\tau}\delta[\mu - \mu_0(w)]\, w^{-2} \frac{\mathrm{d}}{\mathrm{d}w}\left[\frac{1}{1-(1+w^2)}\right]^{1/2}, \qquad (2)$$

with $\tau = [n_e 4\pi e^4 m \ln(\Lambda)/(mc)^3]^{-1}$, $\mu = w_\|/w$, n_r is the instan-taneous runaway density, and n_e is the bulk density.

The Fokker-Planck equation with the source S has been solved analytically [4] for the runaway avalanche growth rate γ_{ra}, and validated against Monte-Carlo and Fokker-Planck calculations. A slightly modified expression obtained from fitting against a Fokker-Planck code is also available [5].

In studying the effect of LH waves on runaway avalanche, the momentum orbit analysis turns out to be very elucidating. Reference 6 developed the trajectories of the secondary electrons which initially lie on the "knock-on" source orbit (dash-dot line in Fig. 1) $w_{\|0} = (1 + w_0^2)^{1/2} - 1$, in the presence of an electric field and small angle Coulomb collisions. They are topologically separated by two separatrices (dashed lines in Fig. 1) defined by the equations of motion. The intersection between the initial "knock-on" orbit and the negative slope separatrix demarcates the minimum momentum ($w_{\|\min}$) above which the secondary electrons will runaway and contribute to the avalanche. It is clear from this picture that the LH waves can effectively modify runaway avalanche if and only if the upper LH resonance boundary (dash-dot-dot line in Fig. 2), $w = (n_{\|\min}^2 w_\|^2 - 1)^{1/2}$, intersects the "knock-on" source orbit trajectory above $w_{\|\min}$ as shown. The process can be modified in two ways.

First, the seed population of the runaway electrons can be significantly enhanced. This can be estimated from previous studies of Dreicer runaway enhancement by LH waves. In Ref. [9], a minimum $\omega/k_\| = v_{\min}$ was imposed and an explicit dependence of the modified

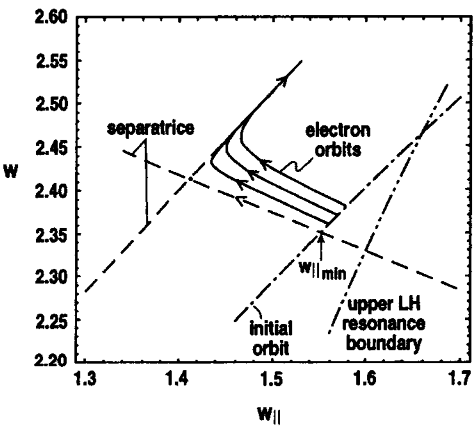

Fig. 1 Momentum orbit trajectories of secondary electrons for $z = 4$, $E/E_c = 2$.

Dreicer runaway flux Γ_D on the wave power and spectrum was derived

$$\Gamma_D \sim \exp\left\{-\frac{1}{2\alpha^2}-\frac{2}{\alpha}-\frac{v_{\min}^2}{2}\right\}, \tag{3}$$

with $\alpha^2 = E(1 + D_0)/E_D$, $E_D = 2\pi e^3 n_e \ln(\Lambda)/T$, and D_0 is the LH quasilinear diffusion coefficient normalized by the collisional diffusion coefficient. The dependence on D_0 is in good agreement with numerical Fokker-Planck calculations. Second, the avalanche growth rate can also be changed. From the momentum space orbit analysis, the growth rate $\gamma_{ra} \propto \int S d^3 w \propto 1/w_{\|\min}$ in the absence of LH waves. In their presence, "knock-out" electrons with momentum as low as the minimum phase velocity (momentum) of the LH spectrum, $v_{\min} = c/n_{\|\max}$ can contribute to the growth rate, depending on the applied power of the LH waves. The maximum enhancement factor for γ_{ra} over the no LH case is thus $w_{\|\min} n_{\|\max}$.

To describe the temporal evolution of the runaway density (and equivalently the runaway current J_{ra}), the two effects are combined in a model to give

$$\frac{dn_r}{dt} = \gamma_{ra} n_r + \Gamma_D . \tag{4}$$

Assuming that γ_{ra} and Γ_D are time independent, the solution at large time is

$$n_{ra} = \frac{\Gamma_D}{\gamma_{ra}} \exp\{\gamma_{ra} t\}, \quad \text{and} \quad \frac{d\ln(J_{ra})}{dt} = \gamma_{ra} . \tag{5}$$

The separation of time scales between the saturation of the Dreicer runaway tail, which takes several collision times, and the avalanche growth time which is much longer, justifies the assumption that Γ_D and γ_{ra} is time independent. To evaluate the accuracy of this model and to explore any variation for $E \gg E_c$, we numerically solve the Fokker-Planck equation with the "knock-on" source.

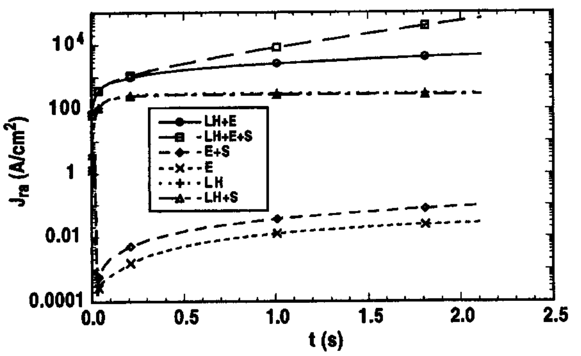

Fig. 2 Runaway currents under different conditions.

3. Fokker-Planck Simulations

The CQL3D Fokker-Planck code [10] is a general purpose code for studying the inter-actions of charged particles via Coulomb collisions and under auxiliary heating conditions to which a "knock-on" source has been added. It solves the bounce-averaged Fokker-Planck equation in velocity space. For the present study, we will neglect particle trapping and focus on the magnetic axis of a tokamak. The LH waves are modeled by a quasi-linear diffusion operator in momentum space

$$\left.\frac{df}{dt}\right|_{ql} = \frac{d}{d\vec{w}} \cdot \left[\vec{D}_{ql} \cdot \frac{df}{d\vec{w}}\right]. \tag{6}$$

For simplicity, we choose a constant D_{ql}.

For this study, we wish to compare the results with those of previous study [5], hence the plasma parameters are chosen to be the same: $n_e = 1 \times 10^{14}$ cm^{-3}, $T_e = 100$ eV. Also, is set to be 1, and it is useful to remember $E_c = 0.102\ n_{14}$ V/m.

Figure 2 presents the runaway current from simulation under different conditions. The lowest two curves are with Ohmic field alone with $E/E_c = 1.5$, without and with the "knock-on" sources, both are very small compared with the Ohmic current and not observable within this time duration. The middle two curves are with lower hybrid turned on at $D_{ql}/D_{coll} = 5$, D_{coll} is the Coulomb diffusion coefficient, $E/E_c = 0$, and with S on and off. Even though the source exists below $v_{\min}$, there is no mechanism to accelerate these particles to the resonant region and beyond, and no avalanche takes place. The resulting current is as predicted by the quasilinear theory of lower hybrid current drive. The second from the top curve is with $E/E_c = 1.5$, LH applied but $S = 0$. The current approaches saturation, is significantly larger than with LH only and represents the LH enhanced Dreicer runaway current [9]. The top curve is with E, LH, and S all turned on. It shows an exponential growth to orders of magnitude larger than the quasilinear prediction within an interval of two seconds.

To compare with the proposed model, Eq. (5) the minimum phase velocity $v_{\min}$ of the LH spectrum is varied while keeping E and D_{ql} fixed, and S turned on as in the top curve of Fig. 2. The relative magnitudes of the current support the picture that lowering $v_{\min}$ results in a larger seed population for the avalanche. Furthermore, the magnitude of the current is found to be self-similar by the factor $\exp\{-v_{\min}^2/2\}$ according to Eq. (4). This is convincing evidence that the amplitude of the runaway current is determined by the LH enhanced Dreicer flux.

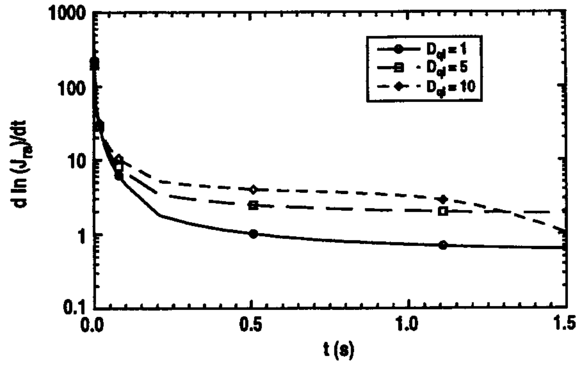

Fig. 3 Dependence of runaway current growth rate on LH power.

To determine whether γ_{ra} depends on the applied LH power, D_{ql}/D_{coll} is varied. Three values, 1, 5, and 10 are plotted in Fig. 3 with the top curve corresponding to the highest D_{ql} value. The growth rate shows a large initial increase with D_{ql}/D_{coll} and saturation as this value approaches 10. The computed enhancement of the growth rate at high LH power compared with no LH waves is approximately 10^2. Both results are consistent with the model put forth using the orbit analysis.

4. Discussion

This study presents the first demonstration that electron runaway avalanche can be significantly enhanced by LH waves. A model of the runaway current, validated by Fokker-Planck calculations, correctly predicts the dependence of the amplitude and growth rate on wave parameters. A corollary result is that the non-inductive current driven by LH waves in the presence of a small Ohmic electric field can be increased by large angle Coulomb scatterings. This offers a new explanation to the LH "spectral gap" issue. Most of the previous explanations had to do with filling the spectral gap with additional waves. In contrast, the new explanation invokes the novel yet classical idea that large angle Coulomb scatterings can accelerate thermal electrons through the gap to high energy. The combined interaction of the source, a small Ohmic electric field, and the lower hybrid waves is capable of significantly enhancing the noninductive current, beyond the quasilinear prediction.

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