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Self-Organized Criticality and Plasma Fluctuation Dynamics

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Abstract

Characteristic self-organized criticality (SOC) [1] dynamics can explain some of the properties of transport in magnetically confined plasmas, and offers a new perspective on how plasma simulations should be carried out. To understand the implications of SOC dynamics in magnetically confined plasmas, new analysis techniques have been developed and applied to the results of numerical simulations. These studies have provided the basis for the application of these techniques to fluctuation data from several confinement devices, including tokamaks, stellarators, and reversed field pinches. The results of the analysis reveal the self-similar character of the edge plasma fluctuations and the existence of long-range time correlations that are consistent with SOC dynamics.

Keywords:
transport, fluctuations, self-organized criticality

Theoretical models of plasma turbulence and induced transport are based on local plasma instabilities. These instabilities have characteristic scale lengths of the order of a few Larmour radii. While radial correlation measurements of plasma fluctuations agree with such scale lengths, local transport coefficients in the L-mode regime do not scale with gyroradius as expected from such local instability models [2]. The so-called Bohm scaling of the plasma diffusivities implies that they carry information on the overall plasma size. Furthermore, perturbative plasma experiments also seem to be inconsistent with local transport properties, and they cannot be modeled by a simple diffusion operator in the transport equation [3].

If we turn our attention to numerical calculations of plasma turbulence, we find that they have often been carried out with constant averaged profiles. In such a situation, transport calculations only show local properties of fluctuations. Such calculations have been useful in investigating the dynamic saturation mechanisms of turbulence, but they fail to present a proper transport picture consistent with the experimental results described above. On the other hand, for critical gradient (or gradient scale length) instability models, when those numerical calculations are carried out with a fixed flux, rather than a fixed gradient, and for long enough times (of the order of transport time scales), changes in the transport properties are indeed observed [4,5]. Local fluctuations at adjacent flux surfaces can be triggered in succession as a result of the quasilinear modification of the averaged plasma profiles, and an avalanche-like transport phenomenon occurs. These avalanches have scale lengths that vary from the fluctuation scale length to the plasma size. They can often be visualized and identified (Fig. 1). They also lead to fluctuation spectra with a 1/f-like region in frequency, and display the plasma behavior that is very close to that observed in a self-organized critical system. Such a spectrum is shown in Fig. 2 at two radial

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positions for a pressure-gradient-driven turbulence calculation [4]. The decay index is about −1.3 for frequencies between 20 and 200, close to −1. Note that the frequency range with a decay index of the order of −1 varies with radial position.

Self-organized criticality (SOC) dynamics [1] may explain some of the properties of transport in magnetically confined plasmas [6,7]. To understand this role of SOC dynamics in plasma transport, it is important to find specific properties that may be used for its experimental identification. One of the main such characteristics is transport by avalanches. This leads to correlation functions with algebraic tails, which signify that long-range time and space correlations exist. Accurately detecting those tails is a problem. For instance, in Fig. 3 the fluctuation autocorrelation function for the same numerical data as shown in Fig. 2 shows a long tail that may have an algebraic decay. The oscillating character of the tail makes it difficult to determine the decay index. This difficulty is further enhanced in the experiment by the presence of noise. Therefore, to experimentally determine such a tail requires new techniques for quantifying the asymptotic behavior of the correlation functions and probability distribution functions. One such technique is the rescaled range (R/S) analysis [8], and the scaled window variance (SWV) technique [9]. These methods determine the Hurst exponent [10], H, which is directly related to the fractal dimension of the time series and to the exponent of the algebraic tail of the autocorrelation function. Applying these techniques to the previously described numerical results and for a range of time lags larger than the decorrelation time of the turbulence, we see that the R/S is a straight line in the log-log plot over more than an order of magnitude in time lag (self-similarity range) with a Hurst exponent of about 0.8 (Fig. 4). This gives us a signature to look for in the experimental analysis.

We have applied the R/S statistics and the SWV technique to the analysis of experimental data. The analysis of electrostatic plasma edge fluctuations for a

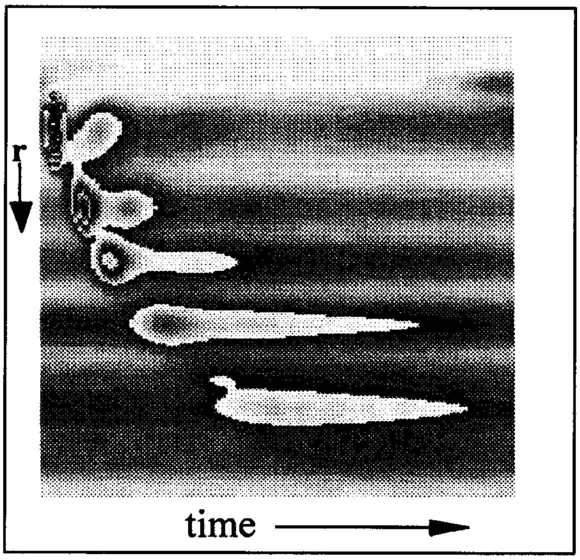

Fig. 1 Avalanche propagation. Sequence of density fluctuations triggered by a single small-size density perturbation from a pressure-gradient-driven turbulence calculation [4].

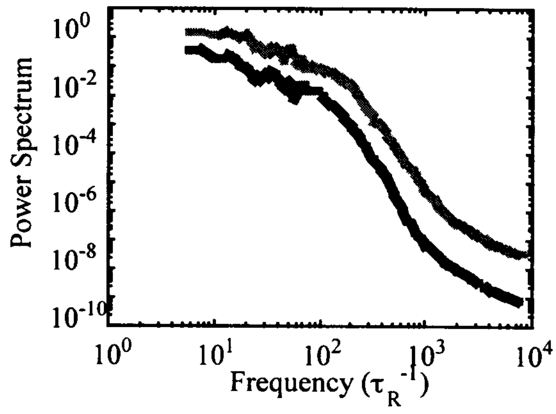

Fig. 2 Density fluctuation spectra at two radial positions of electrostatic potential fluctuations from a pressure-gradient-driven turbulence calculation [4].

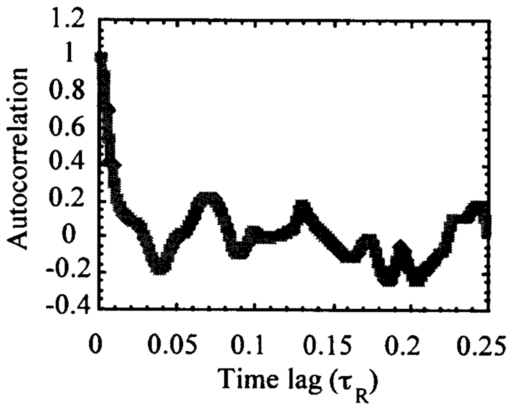

Fig. 3 Autocorrelation function of electrostatic potential fluctuations for the same fluctuation data as Fig. 1.

broad range of magnetic configurations has led to a self-similarity range from the turbulence decorrelation time to a time of the order of the transport time scale. The value of H is between 0.62 and 0.75 [11]. These results are evidence of the existence of long-range correlations in the plasma edge turbulence in those confinement devices. The narrow range of variation of the Hurst exponent suggests a universal character of the plasma edge turbulence dynamics.

Additionally, this narrow range of values for H found in different plasma confinement devices is also an indication of the similarity of the low-frequency range of the fluctuation spectra in those experiments. However, the similarity of the spectra goes beyond the low-frequency range. Using a rescaling transformation of the spectra, we find clear evidence of the similarity of the plasma edge electrostatic fluctuation spectra over the whole frequency range [12].

Some of the techniques used in the detection of the long-range time dependencies can be extended to the detection of cross-correlations for long time lags [13]. They are important because the avalanches should show up as long radial correlation in the long time-lag region. Additionally, an avalanche should show radial propagation. In the resistive pressure-gradient-driven turbulence calculations [4], there was clear evidence of radial propagation. Here, we apply the new diagnostic to the same data to study radial correlations. We detect a rapid decrease of the correlation with increasing radial separation at low time lags, but a much slower decrease is observed at larger time lags. In particular, in the region where the long-range time correlations are significant, the radial correlation is many times longer than the radial correlation length of the fluctuations (Fig. 5). This extension of the analysis techniques allows the determination of radial correlations for noisy signals and, in the case of experimental results, comparison with the radial correlations expected from avalanche transport. Analysis of multichord experimental data is under way [14].

Finally, if long radial correlations in the fluxes exist, they suggest the possibility of a superdiffusive component in the plasma transport [15]. Superdiffusivity may result from both magnetically and electrostatically induced SOC plasma transport. Such a superdiffusive component is consistent with the Bohm-type scaling observed in tokamak plasma transport and can offer an explanation for some of the observed features thought to be inconsistent with theory.

In conclusion, plasma turbulence models when evolved under a fixed flux condition develop some of the characteristic properties of SOC systems. The analysis techniques presented here are effective in detecting these SOC properties when applied to data from these turbulence calculations. This result puts the application of these techniques to experimental data on relatively solid ground.

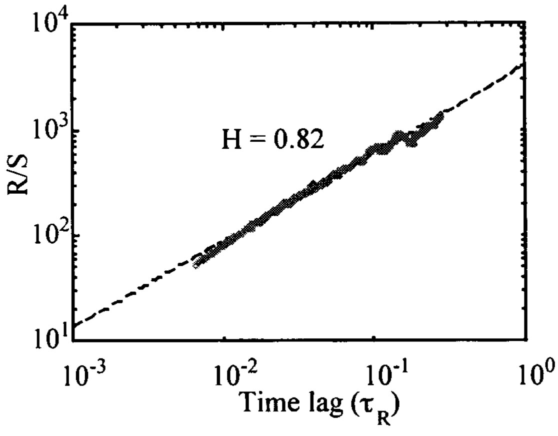

Fig. 4 R/S for the same fluctuation data as Fig. 2 and for values of the time lag above the turbulence decorrelation time.

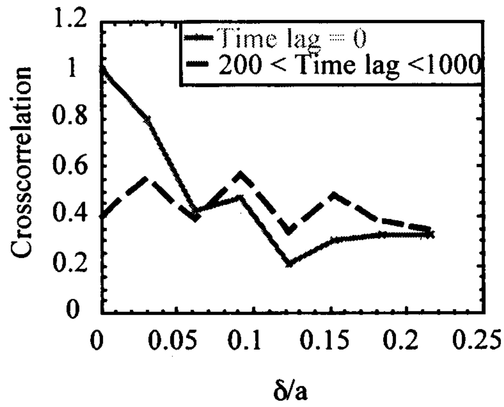

Fig. 5 Radial cross-correlation of the density fluctuations fluctuations as a function of the radial separation. The data are the same as Fig. 2.

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