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Evaluation of Plasma Pressure Profile from Magnetic Measurements in Stellarators

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Abstract

The problem is analyzed whether it is possible or not to determine the plasma pressure profile (or current distribution) from external magnetic measurements in conventional stellarators.

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1. Introduction

It is well known that information obtained from external magnetic measurements is certainly insufficient for getting plasma pressure profile (or current distribution) in tokamaks[1-3]. However, for stellarators more optimistic standpoint has been expressed recently[4-6] with ambitions extending right up to the selection "of the magnetic probe system needed for the determination of both the plasma pressure profile and current distribution"[6].

The main goal of our presentation is to demonstrate the inherent limitations of magnetic diagnostics, which make optimism of Refs.[4-6] groundless. These limitations follow from the first principles, as it was pointed out in Ref.[7]. Now, to make them more clear and tangible, specific examples are considered. They show how and to what extent the difference between pressure profiles becomes masked, when configuration is evaluated on the basis of measurable magnetic signals.

2. What is really measured

First of all, it must be understood that the use of such words as "determination of plasma pressure profile"[6] is incorrect when magnetic diagnostics is discussed. This "determination" is a desire. The reality is the magnetic signals, which can be measured by magnetic loops and coils.

If it is clearly realized, it would be logical to start from the description of the measured values.

Diamagnetic coil, Rogowski loop and magnetic probes located outside the plasma can yield the diamagnetic signal

$$\Delta\Phi = \int_{S_\perp} (\boldsymbol{B} - \boldsymbol{B}_{\mathrm{v}})\, \mathrm{d}\boldsymbol{S}_\perp , \tag{1}$$

the net plasma current

$$J = \int_{S_{\mathrm{p}}} \boldsymbol{j}\, \mathrm{d}\boldsymbol{S}_{\mathrm{p}}, \tag{2}$$

and the magnetic field $\boldsymbol{B}$ and its integral characteristics. The current J belongs to them, and, formally, there is no need to mention it separately. Here $\boldsymbol{B}$ is the magnetic field at the moment of measurements, $\boldsymbol{B}_{\mathrm{v}}$ is the vacuum field, $\boldsymbol{j}$ is the current density, $\boldsymbol{S}_\perp$ is the surface covered by diamagnetic loop, $\boldsymbol{S}_{\mathrm{p}}$ is transverse cross-section of a plasma column.

Imagine the ideal situation: $\boldsymbol{B}$ outside the plasma is completely known after the measurements and proper integration. If the field $\boldsymbol{B}_{\mathrm{ext}}$ due to external currents is also known, plasma-produced field can be found then

$$\boldsymbol{B}_{\mathrm{pl}} = \boldsymbol{B} - \boldsymbol{B}_{\mathrm{ext}}. \tag{3}$$

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This must be considered as the upper theoretical limit for the magnetic diagnostics: $\Delta\Phi$ and $\boldsymbol{B}_{pl}$ outside the plasma. The main question then is what can be deduced from known $\Delta\Phi$ and $\boldsymbol{B}_{pl}$? If plasma pressure is the subject of interest, we should ask: is it possible or not to find the pressure profile by known $\Delta\Phi$ and $\boldsymbol{B}_{pl}$?

It is clear that $\Delta\Phi$ is just a number. But $\boldsymbol{B}_{pl}$ is a function. One can speak about numerous Fourier components of $\boldsymbol{B}_{pl}$ at different locations of the magnetic probes. It seems logical then to conclude that "the more probes, the better". All optimistic opinions and discussions on pressure profile determination[4-6,8,9] are based on this philosophy.

But this philosophy is misleading. It is evidently wrong for a straight cylinder: all measurements would give $B_J = J/(2\pi\rho)$, where ρ is the distance from the axis. Dependence $B_J \sim \rho$ is known in advance, so all possible measurements would give only one useful quantity: the total current J. And no information about pressure, by the way.

The important lesson from this example is that amount of information deduced from the magnetic measurements may be rather limited.

A cylinder with a single "zero" Fourier component of $\boldsymbol{B}_{pl}$ can be criticized as too artificial example. Toroidicity eliminates the degeneracy of a cylinder, and even in a tokamak there is an infinite number of harmonics of $\boldsymbol{B}_{pl}$. But much more harmonics does not necessarily mean much more information.

It is known, which was once more emphasized in Ref.[7], that in equilibrium configuration $\boldsymbol{B}_{pl}$ in the vacuum (outside of the plasma) is determined only by the shape of the boundary Γ and magnetic field $\boldsymbol{B}_\Gamma$ on this boundary. If Γ and near-by magnetic surfaces are circular, $\boldsymbol{B}_{pl}$ is described by two parameters only [7]. In other words, in the infinite set of harmonics only two are independent.

In the case of conventional stellarator two independent values, obtained from measurements of poloidal field, are net current J and "cosine" harmonic of $\boldsymbol{B}_{pl}$ at plasma boundary

$$B_\beta = \int_0^1 \frac{p'(x)}{\mu(x) B_0} x^2 \mathrm{d}x, \tag{4}$$

where p is the plasma pressure, $x \equiv \rho/b$ is the dimensionless radius, b is the minor radius of a plasma, prime stands for the derivative, μ is the rotational transform, B_0 is the toroidal field.

Diamagnetic measurements give in addition

$$\frac{\Delta\Phi}{\Phi_0} = -\frac{\beta}{2} \equiv -\frac{\bar{p}}{B_0^2} = -\int_0^1 \frac{2p}{B_0^2} x \mathrm{d}x, \tag{5}$$

where $\Phi_0 = \pi b^2 B_0$. In (4) and (5) we disregarded the contribution due to the current J, which is usually small, as well as the next-order toroidal corrections.

If there are many probes, a lot of harmonics can be identified. But information found as a result of the measurements may be limited by J and two numbers $\Delta\Phi$ and B_β. It is clear that such a scant information is insufficient for getting $p(x)$.

3. Resolution Imaginary and Real

This "pessimistic" conclusion remains valid in any case. Always the amount of information is limited, because $\boldsymbol{B}_{pl}$ is determined by plasma boundary only [7]. From other hand, different B_β at the same β in two discharges certainly indicate the difference in $p(x)$ which can be characterized by the dimensionless shape factor derived from the measured B_β:

$$S \equiv -\int_0^1 \frac{p'(x)}{\bar{p}} \frac{\mu_b}{\mu(x)} x^2 \mathrm{d}x = -\frac{2\mu_b}{B_0} \frac{B_\beta}{\beta}. \tag{6}$$

Here $\mu_b = \mu(1)$ is the rotational transform at the plasma boundary.

Integral dependence of B_β on pressure profile could be useful for distinguishing different profiles in experiment provided there is a good resolution. In "optimistic" articles the true analysis of this problem is always replaced by a fraudulent trick. Plasma pressure is prescribed as a one-parameter distribution like $p = p_0(1 - x^2)^n$[5,6]. Then it is demonstrated that profiles with different n give different B_β or some other harmonics of $\boldsymbol{B}_{pl}$ (with much smaller amplitudes). But it is never discussed what happens if one of two compared profiles $p(x)$ does *not* belong to the analyzed family.

Such comparison would immediately destroy the optimism about determining plasma pressure profiles from magnetic measurements. When artificial restrictions on permissible profiles are eliminated, the seeming "resolution" disappears. To show it, we consider below the extreme case of "no resolution", when given pair ($\Delta\Phi$, B_β) corresponds to different $p(x)$.

Let us start from two arbitrary pressure profiles $p_1(x)$ and $p_2(x)$ corresponding to β_1 and β_2. It is clear that profiles $p_1(x)$ and $(\beta_1/\beta_2)p_2(x)$ have the same β and would give the same diamagnetic signals. Deformation of the profile

$$p(x) \longrightarrow p(x) - D\delta(x) \qquad (7)$$

does not change β if

$$\int_0^1 \delta(x) x \mathrm{d}x = 0. \qquad (8)$$

This property can be used for changing B_β, while $\Delta\Phi$ is kept fixed. If constant D is chosen from the condition

$$\frac{\beta_1}{\beta_2}\int_0^1 \frac{p_2'}{\mu} x^2 \mathrm{d}x - D\int_0^1 \frac{\delta'}{\mu} x^2 \mathrm{d}x = \int_0^1 \frac{p_1'}{\mu} x^2 \mathrm{d}x, \qquad (9)$$

profiles $p_1(x)$ and

$$p_3(x) = \frac{\beta_1}{\beta_2} p_2(x) - D\delta(x) \qquad (10)$$

would give the same pair ($\Delta\Phi$, B_β).

In contrast to the case of the prescribed "one parameter set of plasma pressure profiles"[4-6], this algorithm leaves a great freedom. Instead of only one profile with given pair of measurable values, as proposed in [4-6], there is a wide family of such profiles.

To construct them by using Eq.(10), first we must find proper $\delta(x)$. Equation (8) is satisfied by any

$$\delta = \frac{1}{x}\frac{\mathrm{d}Y}{\mathrm{d}x} \qquad (11)$$

with $Y(0) = Y(1)$. It is natural to assume that plasma pressure vanishes at the boundary. It gives an additional constraint for δ:

$$\delta(1) = 0. \qquad (12)$$

All these requirements are met by

$$Y = x^2(1-x)^2 Z(x). \qquad (13)$$

Here $Z(x)$ is an arbitrary function without singularities. Note that δ is finite at $x = 0$ due to x^2 in Eq.(13).

For illustration let us take $Z(x) = x^2$ and two clearly different profiles with the same β (in normalized units)

$$p_1 = \frac{3}{4}(1 - x^4), \qquad p_2 = 1 - x^2. \qquad (14)$$

Also, assume that profile of the rotational transform is parabolic:

$$\mu(x) = \mu_0 + (\mu_b - \mu_0)x^2. \qquad (15)$$

Here μ_0 and μ_b are the values of the rotational transform at the axis and boundary, respectively.

For this case the formal solution of Eq. (9) for the "deformation coefficient" D is shown in Fig. 1. In Heliotron E $\mu_0/\mu_b = 0.5/2.5 = 0.2$, then $D = 1.324$ and

$$p_3 = 1 - x^2 - 2.65x^2(1-x)(2-3x). \qquad (16)$$

This profile together with p_1 is shown in Fig. 2. Difference between $p_1(x)$ and $p_3(x)$ is great and certainly can be detected experimentally, when temperature and

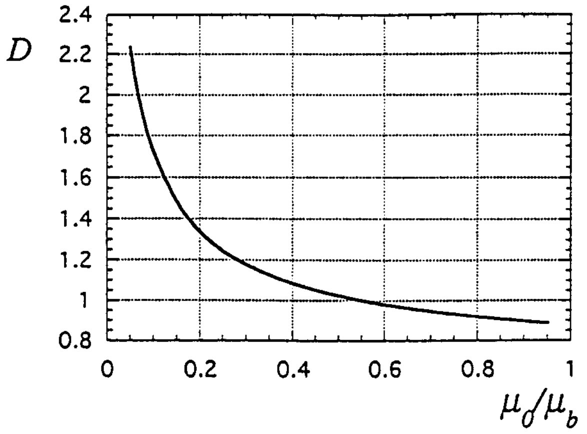

Fig. 1: D as a function of μ_0/μ_b for pressure profiles (14), rotational transform (15), and $Y = x^4(1-x)^2$.

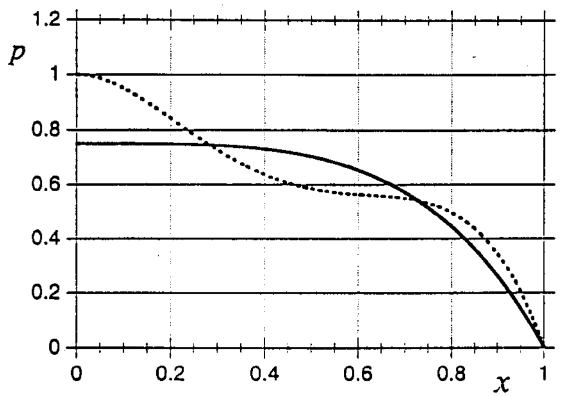

Fig. 2: Two pressure profiles (normalized units) with the same pair of measured values ($\Delta\Phi$, B_β) for Heliotron E: full curve $p = 0.75\,(1 - x^4)$; dashed curve $p = 1 - x^2 - 2.65x^2(1-x)(2-3x)$.

density are measured. But this difference cannot be seen by means of magnetic measurements, because both profiles would give the same pair of $(\varDelta\Phi, B_\beta)$.

4. Discussion

Integral nature of magnetic measurements does not allow to find internal distributions. It must be considered as a general rule without exceptions. Certainly, measurable values give some information about pressure and current profile. But in practical situations the amount of information is strongly limited by several independent values only.

All proposals to distinguish different profiles are in fact based on a masked substitution "number → profile". However, the true interpretation must be "number → family of profiles". Profiles belonging to such rather wide family cannot be distinguished by means of magnetic measurements. They can differ very much, which is shown above.

It must be noted that it was easy to construct two clearly different profiles with the same pair of measurable values $(\varDelta\Phi, B_\beta)$. This easiness is the manifestation of the great freedom (or degeneracy) still remaining within two constraints, $(\varDelta\Phi, B_\beta)$. It cannot be suppressed strongly in more complicated models with 3 or 4 independent measurable magnetic parameters corresponding to the better account of toroidicity or another plasma shape.

Finally, only rough guess at the pressure profile (or current distribution) can be done on the basis of external magnetic measurements.

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