

RMPを加えた領域における不純物輸送シミュレーションのための
モンテカルロ・ポアソンソルバー開発

Development of Monte-Carlo method solving Poisson equation in transport simulation of impurities affected by RMPs

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Accumulation of tungsten impurities in a core of toroidal plasma is one of the most critical issues for operating fusion reactors. At present, studies of the impurity transport in the edge region are not sufficient for assessment of the accumulation level. Several studies [1,2] have reported recently that Coulomb collisions have major impact on the impurity transport. Accordingly, in this study, we focus on the collisional transport of the impurities.

Recently, resonant magnetic perturbations (RMPs) are often used for mitigating edge localized modes (ELMs). In this case, the magnetic structure in the edge region becomes 3-dimensional (3D), and thus the impurity transport is affected by the electric field in addition to the 3D magnetic field. The electric field can be given by solving the Poisson equation, but required performance of the Poisson solver is not simple because the solver is applied to the complicated structure of the charge density of the background ion (hydrogen) and electron affected by the RMPs. Furthermore, since the impurity transport in such a structure is the problem in this study, the electrostatic potential is desired to be calculated only in the edge region, and not be calculated all over the core. For the reasons mentioned above, we are developing a new Monte-Carlo method solving the Poisson equation.

The method is based on the following theorem [3].

Theorem: Define a function $u(\mathbf{x})$ as follows.

$$u(\mathbf{x}) = E^{\mathbf{x}}[G(\mathbf{x}(\tau))] - E^{\mathbf{x}} \left[\int_0^{\tau} F(\mathbf{x}(t)) dt \right],$$

where $E^{\mathbf{x}}$ is the expectation operator of a stochastic process $\mathbf{x}(t)$ given by a stochastic differential equation:

$$d\mathbf{x}(t) = \sqrt{2}d\mathbf{W}_t.$$

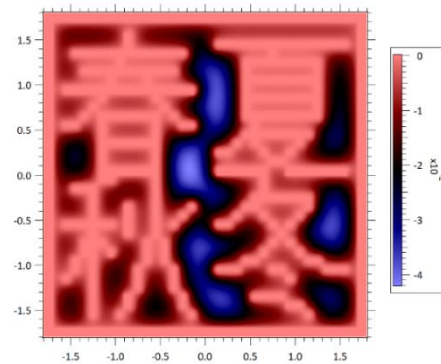
Here, the process $\mathbf{x}(t)$ starts from \mathbf{x} at time $t = 0$, \mathbf{W}_t is the Wiener process, and τ is the first exit time from the domain of the process $\mathbf{x}(t)$. Then $u(\mathbf{x})$ is the unique solution of the Poisson equation:

$$\Delta u(\mathbf{x}) = F(\mathbf{x}) \text{ in Cartesian coordinates,}$$

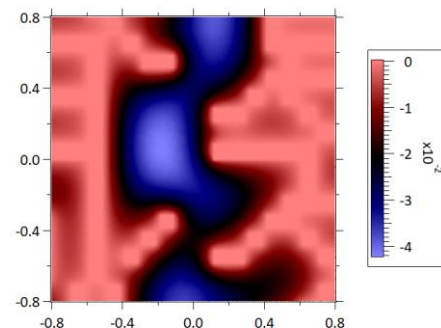
where the boundary condition is defined as

$$u(\mathbf{x}) = G(\mathbf{x}) \text{ at the boundary.} \quad \square$$

For a benchmark of the Poisson solver, we apply it to a 2-dimensional example having a complicated boundary given by KANJI characters “春夏秋冬” with the conditions of $F(\mathbf{x}) = 1$ and $G(\mathbf{x}) = 0$. The potential $u(\mathbf{x})$ is given as the following figure.



Keeping the conditions of $F(\mathbf{x})$ and $G(\mathbf{x})$ all over the domain, this solver can be also applied only to a part of the domain for calculating the potential $u(\mathbf{x})$ in the part, as shown in the following figure. The development of the solver is successfully in progress.



- [1] Y.Homma *et al.*, Nucl. Fusion **56** (2016) 036009.
- [2] R.Kanno *et al.*, Nucl. Fusion **60** (2020) 016033.
- [3] A.Friedman, *Stochastic Differential Equations and Applications* (Dover, 2004).