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Dynamic Mode Decomposition of Measurement Data in Tokamak Device PHiX

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1. Research Background

Dynamic mode decomposition (DMD) [1] is a powerful data-driven tool for extracting a reduced-order model of a non-linear dynamical system. DMD extracts a low-rank structure of the system in the form of spatial modes and its corresponding temporal dynamics. The modes are constructed by the eigendecomposition of an approximated linear operator that advances the spatiotemporal learning dataset in time. The linear nature of the approximated operator allows one to extrapolate the time evolution beyond the learning dataset hence DMD is also often used as a forecasting algorithm for a non-linear dynamical system. DMD is often considered as a numerical approximation of the Koopman spectral analysis which relates the approximated linear operator in DMD to Koopman Operator. Lange et al. [2] compares the performance of Koopman spectral analysis with the Fourier method and neural-network-based long short-term memory (LSTM) in long-term forecasting of temporal signals which shows the strength of Koopman spectral analysis over other methods.

DMD is revitalized in recent years due to the surge of data and computational performance. Predominantly used in fluid dynamics, DMD began to be applied to various research fields. Sasaki et al. [3] used DMD in cylindrical magnetized plasma to extract the limit cycle of Kevin-Helmholtz turbulence. Kaptonoglu et al. [4] used DMD to analyze and forecast magnetic probe signals in the HIT-SI experiment. In this research, DMD is used for camera data and other measurement data analysis in the small tokamak device PHiX in Tokyo Tech.

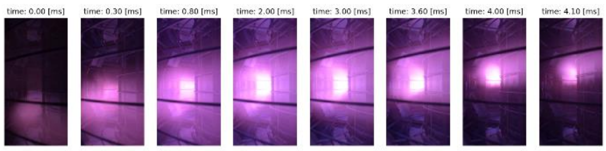

Fig. 1. RGB footage of high-speed camera (10^5 fps) in PHiX

2. Dynamic Mode Decomposition

Let a spatiotemporal sequence data be denoted as $\boldsymbol{x}(t_n) \in \mathbb{R}^{N_x}$ where N_x is the number of spatial nodes or measurements and t_n is the temporal discretization or sampling time. In term of camera footage, $\boldsymbol{x}(t_n)$ is the intensity of all pixels in a frame reshaped into one-dimensional array. Eigendecomposition of DMD is given by

$$\boldsymbol{x}(t) \approx \boldsymbol{x}^{\mathrm{DMD}}(t) = \sum_{j=0}^{r-1} b_j \boldsymbol{\varphi}_j e^{(\delta_j + i\omega_j)t},$$

and calculated by singular value decomposition-based method. r is the DMD rank and the total number of the DMD modes. Here, red channel of the camera footage of PHiX is analyzed with 12-rank DMD and 0.12 of relative Frobenius error is obtained. Several dominant DMD modes can be seen in the following figure. DMD modes are generally complex valued and often come in conjugate pairs. Only one of each pair is shown in Fig. 2.

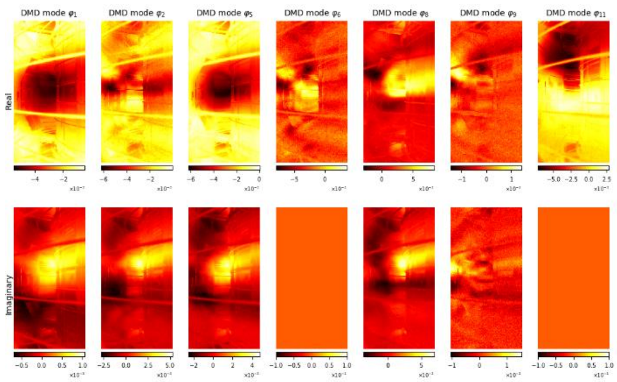

Fig. 2. Several dominant DMD modes from the red channel of PHiX camera footage.

References

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[4] A.A. Kaptanoglu et al., *Characterizing magnetized plasmas with dynamic mode decomposition*, Phys. Plasmas, **27** (2020), 032108