

Plasma parameter profile inference from limited data utilizing second-order derivative priors and physic-based constraints

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A Bayesian framework has been used to improve the quality of plasma parameter profile inference. An integrated data analysis (IDA) allows for coherent combinations of different diagnostics[1], and the Gaussian process (GP) regression provides a reliable regularization procedure and systematic uncertainty estimation[2]. We propose a new profile inference framework that utilizes our prior knowledge about plasma physics, along with integrated data analysis and the Gaussian process.

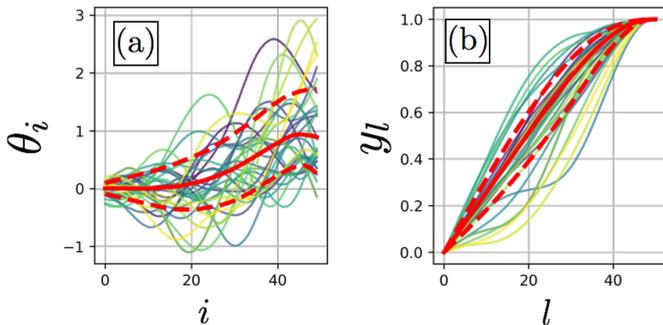


Fig. 1: Original samples (a) and corresponding profiles (b).

As demonstration and validation of the proposed inference framework, we consider a one-dimensional system composed of a hydrogen plasma and hydrogen atoms, where electron particle transport is described by a diffusive process with a diffusion coefficient D_{n_e} . When we assume the system is quasi-stationary, divergence of a particle flux is balanced by a particle source S . Thus, the prior knowledge for the electron particle transport can be incorporated

into a probability distribution as follows:

$$P(I_{pb}|\mathbf{n}_e, \mathbf{n}_0, \mathbf{T}_e) \propto \exp\left(-\int_{L_{min}}^{L_{max}} \frac{\left(\frac{\partial}{\partial L}(D_{n_e} \frac{\partial}{\partial L} n_e) + S\right)^2}{2n_e^2 \hat{\tau}^{-2}} dL\right), \quad (1)$$

where \mathbf{n}_e , \mathbf{n}_0 , and \mathbf{T}_e are the electron density profile, the neutral profile, and the electron temperature profile, respectively. I_{pb} stands for the prior knowledge. $\hat{\tau}$ is a hyper-parameter, which specifies the weight of this prior. In addition, we introduce another prior using a GP. While the posterior distribution can be obtained analytically if the data is linear with respect to the parameters of interest, we formulate the inference framework using a Markov Chain Monte Carlo method (MCMC). This allows us to incorporate diagnostics such as Balmer-line intensities into IDA. In order to reduce the correlation between parameters and facilitate the use of MCMC, we construct profiles by integrating the original parameters twice as shown in Fig. 1. The samples in the original parameter space are drawn from a GP using MCMC. In addition, a monotonicity condition is imposed by adding a penalty term. We can see that the Markov chain relatively freely explores the original parameter space.

Next, we simultaneously infer the profiles of n_e , T_e , n_0 , and D_{n_e} using the priors based on the electron particle conservation and a GP. We assume that available diagnostics are the line-integrated absolute intensities of four Balmer lines (D_ϵ , D_δ , D_γ , and D_β), and the line-integrated electron density. The synthetic data for those measurements are gen-

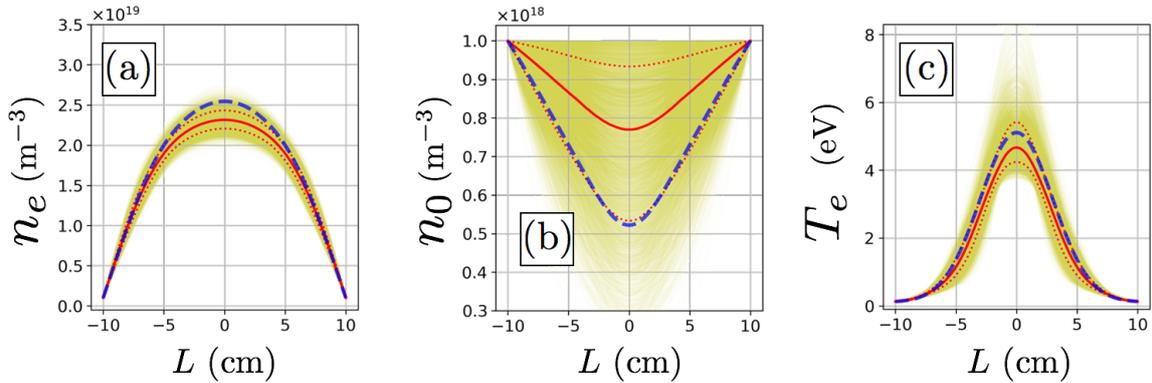


Fig. 2: Results of n_e (a), n_0 (b) and T_e (c) inference from Balmer-line intensities and line-integrated electron density. The input profiles are shown in the blue dashed lines. The red dotted lines represent the 16th and 84th percentiles at each spatial point. The red solid lines represent the 50th percentiles at each spatial point. The original samples are shown in the yellow transparent lines. $\hat{\tau} = 1$ ms is used.

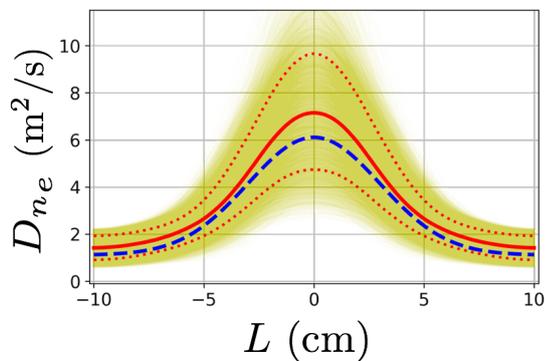


Fig. 3: Results of D_{n_e} inference. The input profiles are shown in the blue dashed lines. The red dotted lines represent the 16th and 84th percentiles at each spatial point. The red solid lines represent the 50th percentiles at each spatial point. The original samples are shown in the yellow transparent lines.

erated from a one dimensional plasma. Here, the diffusion coefficient for n_0 is set to $100 \text{ m}^2/\text{s}$ while $D_{n_e} = 1 + T_e \text{ m}^2/\text{s}$. (The unit for T_e is eV). When inferring the profiles, we further assume that the D_{n_e} profile is given by:

$$D_{n_e} = a_{D_{n_e}} \mathbf{1} + b_{D_{n_e}} T_e. \quad (2)$$

But, the exact values of $a_{D_{n_e}}$ and $b_{D_{n_e}}$ are unknown.

The results of the n_e , n_0 , and T_e inference are shown in Fig. 2. We can see that localized distributions of n_e and T_e can be obtained only from line-integrated measurements. A useful estimate of n_0 is also obtained. Since this inference provides

T_e , $a_{D_{n_e}}$, and $b_{D_{n_e}}$, we can obtain the probability distribution of D_{n_e} shown in Fig. 3.

The proposed analysis framework allows for meaningful inference of plasma parameter profiles even when only line-integrated quantities can be measured. Furthermore, the physics-based information incorporated into the prior offers the inference of the D_{n_e} profile, to which no diagnostic used in the measurement is sensitive.

References

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