S8-5

Progress in Magnetic Field Coil Optimization for Stellarator Plasmas

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Finding an easy-to-build coils set has been a critical issue for stellarator design for decades. There are two main categories of methods to optimize the external coils for producing the target MHD equilibrium. The first one solves a surface current using Green's functions and approximate discretized coils subsequently. The other one represents coil explicitly and optimize coil filaments considering engineering constraints. We will introduce these methods and particularly present the development of the new coil design code, FOCUS.

1. Introduction

The concept of stellarator optimization separates the problem of designing a high-performance plasma from the task of designing a suitable coils set. First, an attractive plasma equilibrium is identified by using a nonlinear optimization. The second stage of the design, which is the topic of this presentation, is to design a set of discrete current-carrying coils that creates the required external 'vacuum' field for confining the 'reference plasma' configuration.

The vacuum field, $\boldsymbol{B_V}$, by which we mean the magnetic field produced by currents external to the plasma domain, must balance the magnetic field, $\boldsymbol{B_P}$, produced by the currents that may or may not be present in the plasma, so that the normal component of the total magnetic field, $\boldsymbol{B} = \boldsymbol{B}_V + \boldsymbol{B_P}$, on S is zero. For vacuum fields, the Ampere's Law in ideal MHD equations reduces to $\nabla \times \boldsymbol{B_V} = 0$. Together with the magnetic divergence constraint $\nabla \cdot \boldsymbol{B_V} = 0$ we can easily derive $\nabla^2 \phi = 0$, where ϕ is the magnetic scalar potential. Thus, the coil determination problem is to solve Laplace's equation with the boundary condition of $\boldsymbol{B}_V \cdot \boldsymbol{n} = -\boldsymbol{B_P} \cdot \boldsymbol{n}$ on S, where n is the unit surface normal.

Beyond the needs of producing target magnetic field, coils are also required to be easy-to-build. This involves several engineering constraints that coils should meet. Since 1980s, Tremendous efforts have been made to find optimal coils that meet both the 'physics' requirements and the 'engineering' requirements.

2. Surface Current Approximation

Pioneering work in the field of coil design was performed by Merkel with the development of the NESCOIL code [1], in which he assumed that the external magnetic field is produced by a surface current distribution on a closed toroidal surface surrounding the plasma. This toroidal surface constrains the location of the coils and is called the 'current carrying surface' or the 'winding surface'. The surface current density, $\mathbf{j} = \mathbf{n} \times \nabla \Phi$, is expressed by a current potential, Φ, on the winding surface. A Green's function method is then applied to solve for the current potential distribution that minimizes the squared normal error. Once the surface current potential is determined, a set of discretized coils can be obtained by selecting an appropriate number of contours of Φ. Merkel's method leads to a Neumann condition problem that can be linearly solved, and it's inherently fast and robust. A singular value decomposition (SVD) method [2] and, more recently, a Tikhonov regularization approach [3] were applied to provide improvements on NESCOIL.

3. Nonlinear optimization methods

A different approach that explicitly incorporates engineering constraints has been advanced by Drevlak with the extended NESCOIL code and ONSET [4], by Strickler *et al.* with the code COILOPT [5], and later by Breslau *et al.* with COILOPT++. The coils are represented as 'filaments', one-dimensional curves, lying on a toroidal winding surface (which is either pre-defined or optimized simultaneously). The magnetic field produced by δ-function current densities in the coils set is calculated using the Biot–Savart law. The geometry of the coils is varied using nonlinear optimization algorithms to minimize a 'cost-function' that represents a balance between the physics requirements (that the total normal magnetic field at the plasma boundary is as small as possible) and the engineering constraints (that the coils can be achieved by modern engineering techniques).

4. FOCUS

4.1 *Three-dimensional representation*

For all the methods mentioned above, a toroidal winding surface is required to locate the coils (ONSET may need two constraining surfaces for interpolation). However, a 'bad' winding surface directly results in the failure of finding an acceptable coils set. Here, we present a new method for designing stellarator coils that eliminates the winding surface altogether [6].

We employ a very easy way of describing one-dimensional curves embedded in three-dimensional space. A curve is described directly, and completely generally, in Cartesian coordinates as $\mathbf{x}(\mathrm{t}) = \mathrm{x}(\mathrm{t})\,\mathbf{i} + \mathrm{y}(\mathrm{t})\,\mathbf{j} + \mathrm{z}(\mathrm{t})\,\mathbf{k}$. Three functions are required to specify the geometry, namely x(t), y(t) and z(t), with the constraints that each function be periodic, e.g. $\mathbf{x}(\mathrm{t} + \mathrm{T}) = \mathbf{x}(\mathrm{t})$ for some T. The curve parameter, t, at this point is arbitrary. A variety of mathematical representations are possible. For purpose of illustration, and because our initial interest is in smooth coils, we use a Fourier representation,

$$x(t) = x_{c,0} + \sum_{n=1}^{N_F} [x_{c,n} \cos(nt) + x_{s,n} \sin(nt)] \quad (1)$$

with t varying between $[0,2\pi]$, and similarly for y(t) and z(t). The shape of a coil is then fully determined by the $3\times(2N_F + 1)$ Fourier coefficients. No additional assumptions are made here, so this representation fits for all kinds of closed smooth coils, such as helical, modular, saddle, etc.

4.2 *Objective functions*

The coil parameters are to be varied to minimize a target function consisting of both 'physics' and 'engineering' objective functions

$$\chi^2(\boldsymbol{X}) = \sum w_j (\frac{f_j(\boldsymbol{X}) - f_{j,o}}{f_{j,o}})^2 \quad (2)$$

where $f_j(\boldsymbol{X})$ is the value of the j-th objective function, to be defined below, for a given set of coil parameters, $f_{j,o}$ denotes the desired value, and w_j is a user-prescribed weight.

The following cost functions have been implemented in FOCUS: 1) normal field error, 2) toroidal flux error, 3) magnetic field Fourier components, 4) coil length, 5) coil to coil separation and 6) coil to plasma separation.

The derivatives of the weighted target function, χ^2, can be computed analytically by using functional derivatives [7]. Therefore, FOCUS can employ powerful minimization algorithms, without approximating the gradient or Hessian.

4.3 *Applications*

FOCUS has been validated by reproducing the W7-X modular coils starting from an arbitrarily circular initialization and applied to multiple configurations for different types of coils, including modular coils, helical coils, trim coils, etc. With analytically calculated Hessian, FOCUS is able to use eigenvalues of the Hessian matrix for determining the error field sensitivity to coil deviations [8]. The sensitivities provide information to avoid dominant coil misalignments and simplify coil designs for stellarators. A proof-of-principle example is given on a CNT-like configuration. More practical applications are conducted on the CFQS stellarator that is being built in China.

5. Conclusions

Different coil optimization methods have distinct features. FOCUS has shown improvements on simplifying the optimization procedure, accelerating nonlinear optimization and error field sensitivity analysis. For future stellarators, a combination of different tools would be preferable.

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