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Quasi-symmetry and optimization in stellarator plasmas

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1. Introduction

The toroidal magnetic confinement needs the rotational transform, because the twisted magnetic field line can cancel the charge separation due to the ∇B drift of the ion and electron. Therefore, the toroidal magnetic confinement can be classified by the method how to make the rotational transform. One is a most famous, the tokamak, the toroidal current is used to produce the rotational transform. The tokamak configuration can be geometrically symmetric along the toroidal direction, that is the axisymmetric configuration. This symmetry leads many advantages. One is the good confinement property of the particle orbit. From the Hamiltonian dynamics, the canonical momentum can be defined and then the perfectly closed the orbit surface can be guaranteed. On the other hand, another magnetic configuration is the stellarator. An important characteristic is noted that the rotational transform is produced by the 3D shaping of the flux surface. That means the magnetic configuration is intrinsically and geometrically no symmetric, that is, the non-axisymmetric. In the stellarator, because of no symmetry, the canonical momentum cannot be defined and then the transport degrades comparing with the tokamak. This problem was a long standing issue for the stellarator research. However, the quasi-symmetry configuration was found to solve this problem. In this talk, the history of stellarator optimization studies are reviewed briefly and recent progresses of stellarator optimizations are discussed.

2. Brief history of stellarator optimization

Stellarator optimization studies had been started in 1970's in Garching, Germany. In Garching, classical stellarators, Weldelstein stellarators, were operated but those machines were facing serious problems of the plasma performances. Because, the Wendelstein stellarator was a classical low magnetic shear stellarator, the achieved plasma beta value was very low. Results of the Wendelstein could not extrapolate to the reactor comparing with the tokamak. Therefore, the stellarator optimization study had been started to design the high beta stellarator. In Wendelstein stellarator, the achieved beta value was limited by the Shafranov shift, because the Shafranov shift made serious MHD events on the low order rational of the low magnetic shear. The Shafranov shift is driven by the plasma equilibrium current, that is, the Pfirsch–Schlüter (P-S) current. In addition, the magnetic well depth was not deep.

The P-S current can be suppressed by the high elongated plasma shape. To produce the high elongated plasma shape, the magnetic field line is strongly modulated with the helical magnetic axis. Figure 1 shows an example of the optimized stellarator for the high beta equilibrium and stability.

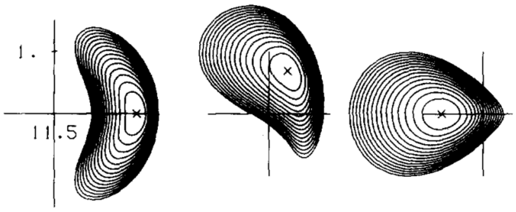

Fig.1 Flux surfaces of the high beta stellarator

This optimized stellarator is so-called "Helias: Helical axis Advanced Stellarator". In fig.1, one characteristic is clearly shown, that is, flux surfaces are changing from the bean shaped cross section to, the teardrop shaped cross section, and the triangular shaped cross section. The bean shaped cross section is necessary to elongate the flux surface strongly. This strong elongation makes strong reduction of the P-S current. In this cross section, the magnetic field line is strongly twisting. But, from the teardrop shaped cross section to the triangular shaped cross section, the magnetic field line moves almost straight in the high field side. Thus, the effective magnetic field is increased from the axis to the plasma boundary. That nature can make deep magnetic well. In a theoretical prediction, this optimized configuration is possible at $\langle\beta\rangle$~10%.

Most important point is a nonlinearity of spectrum of the plasma boundary shape and magnetic field. Equation 1 shows a Fourier representation of the plasma boundary shape, *r* and *z*, proposed by Garbedian,

$$r + iz = e^{iu} \sum \Delta_{mn} e^{-imu+inv} \, . \qquad (1)$$

Here, *m* and *n* are poloidal and toroidal mode numbers of Fourier spectrum. The *u* and *v* are poloidal and toroidal angles. If spectrum of the plasma boundary and magnetic field are compared, a relation of both spectrum is not linear. This nature leads an important conclusion, that is, spectrum of the magnetic field can be manipulated by spectrum of the plasma boundary shape.

3. Quasi-symmetry and omnigeneity of magnetic field

The Helias configuration was realized as the Wendelstein 7-AS stellarator and good high beta equilibrium and stability properties were confirmed. However, the orbit property, which leads the transport, was a serious problem, because very sophisticated 3D shaping of the plasma boundary, in other words, large helical ripples. Usually, if the symmetry of the magnetic field exists, the canonical momentum can be defined. In such a case, the perfectly closed orbit surface can be assumed, in other words, the particle can be confined. However, because of no symmetry in the stellarator, the canonical momentum cannot be defined. To improve the orbit property, spectra of the magnetic field were manipulated and the quasi-helical symmetric configuration was found. In figure 2, spectra of the magnetic field in a quasi-helical symmetric stellarator is shown.

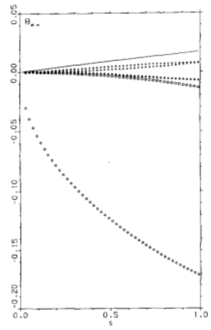

Fig.2 Spectra of the magnetic field for a quasi-helical symmetric configuration

In the quasi-helical symmetry, although the magnetic field is completely the 3D, but the dominant mode of spectra of the magnetic field is only the helicity. That is almost (not perfect) equivalent to the straight helical configuration. Because of the “quasi”-helical symmetry, the canonical momentum cannot be defined, but the closed 2nd adiabatic invariant surface can be assumed. Therefore, the orbit property is significantly improved. After the finding the quasi-helical configuration, other symmetries, the quasi-axisymmetric and quasi-poloidal symmetric configurations were found. In particular, the quasi-axisymmetric configuration is possible to design the low plasma aspect ratio and the intermediate configuration of the stellarator and tokamak.

Finally, a recent topic of the quasi-symmetric configuration is introduced. Up to now, the optimization study with the quasi-symmetry is done by the computations. Of course, at the beginning, the configuration was designed analytically but the wide range scan of the configuration space needs the huge computation. However, recently, the theory of the quasi-symmetry is developed and some good quasi-symmetric configurations are found. An interesting point is the omnigenity is a natural extension of the quasi-symmetric configuration. In figure 3, the concept of this idea is shown.

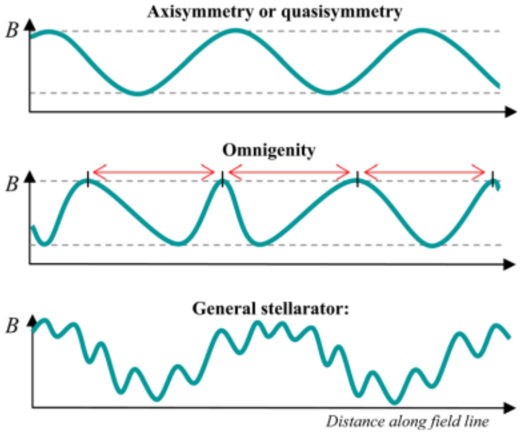

Fig.3 Omnigenous fields have an intermediate level of complexity between quasisymmetric and general nonaxisymmetric fields.

The quasi-symmetry can be defined easily the constant 2nd adiabatic invariant, that is, closed contours of invariants. However, to defined closed contours of invariants, the symmetry is not necessary. In the meddle of figure 3, constant invariants can be defined without the quasi-symmetry. That is another way to the stellarator.