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直線プラズマにおける乱流解明を目指した特異値分解法を用いたトモグラフィアルゴリズムの開発
Development of SVD Algorithm for Turbulence Tomography in Linear Plasma

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Plasma turbulence is one of the important issues to be studied for realizing a nuclear fusion reactor. It is because plasma turbulence degrades the plasma confinement property greatly. At present, for further understanding of the plasma turbulence, it is necessary to observe the turbulence not only in local region of plasma but also over the entire plasma. Tomography is one of possible diagnostics to make such measurements in sufficient spatial and temporal resolution.

We have constructed a prototype of such tomography system to measure plasma local turbulence for a linear cylindrical device, Plasma Assembly for Nonlinear Turbulence Analysis (PANTA) in Kyushu University. We are now developing a tomography algorithm using Singular Value Decomposition (SVD) in addition to the already developed ones, Maximum Likelihood-Expectation Method (MLEM), Algebraic Reconstruction Technique (ART), Fourier-Bessel series and Cormack expansion.

The purpose of our tomography is to obtain plasma local emissivity from a set of the line-integrated values of the emission. The relation between the line-integrated values g_i and local emission ε_i is expressed as

$$g = h\varepsilon \qquad (1)$$

where h is a matrix composed of the areas of the line-of-sights included in the grids as is shown in Fig.1. The emission can be obtained with a least square method to minimize the modified x^2 function, with assuming $\| \|$ being the norm,

$$\Phi = \| g - h\varepsilon \|^2/M + \Upsilon \| C\varepsilon \|^2 \qquad (2)$$

where Υ is the regularization parameter and C is selected to be the identify matrix here. The solution is expressed as

$$\varepsilon = (h^T h + M\Upsilon C^T C)^{-1} h^T g \qquad (3)$$

The solution can be rewritten using the SVD, which factorizes a matrix to three elemental products, and $hC^{-1} = UWV^T$ where U and V represent orthogonal matrices, and W represents diagonal matrix composed of singular values.

$$\varepsilon = \sum_{i=1}^{M} w_i(\Upsilon) \frac{g \cdot u_i}{\sigma_i} C^{-1} v_i, \; w_i(\gamma) = 1/(1 + M\gamma\sigma_i^{-2}) \qquad (4)$$

where σ_i represents *i-th* singular value, u_i and v_i are *i-th* column vector of matrices *U* and *V*.

Figure 2 shows an example of the reconstructed images using SVD method and MLEM for comparison, together with residual error and MAE (Mean Absolute Error). The definition of MAE is

$$MAE = \frac{1}{N} \sum_{j=1}^{N \times N} \left| \varepsilon_j - \hat{\varepsilon}_j \right| \qquad (5)$$

where $\hat{\varepsilon}$ represents the original value. Note that here we set $\Upsilon = 10^{-7}$. The assumption for the calculation is that the plasma is observed with totally 1616 detectors. The detectors are assumed to be located at 16 different azimuthal angle positions with 101 channels for each direction, respectively. In the original image, there is a peak on the left-top to examine whether the azimuthally asymmetric structure can be reconstructed or not. The results show that the sharp peak can be reconstructed on both methods. In this case, the total error of the SVD is smaller than that of the ML-EM.

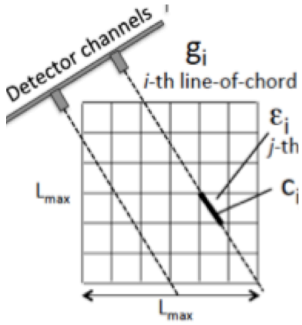

Fig.1. Schematic view of tomography condition

In summary, tomography algorithm based on SVD is being developed, and its application to an assumed emission data well reproduces the original image well. As a future work, the best regularization parameter Υ also should be selected using GCV criteria [1,2], and comparisons between the SVD and other already developed algorithms should be made in aspects of reproducibility, computing speed, and so on. In this conference, we report the results of tomography reconstruction using the SVD algorithm, and its real application on the experimental data obtained in PANTA.

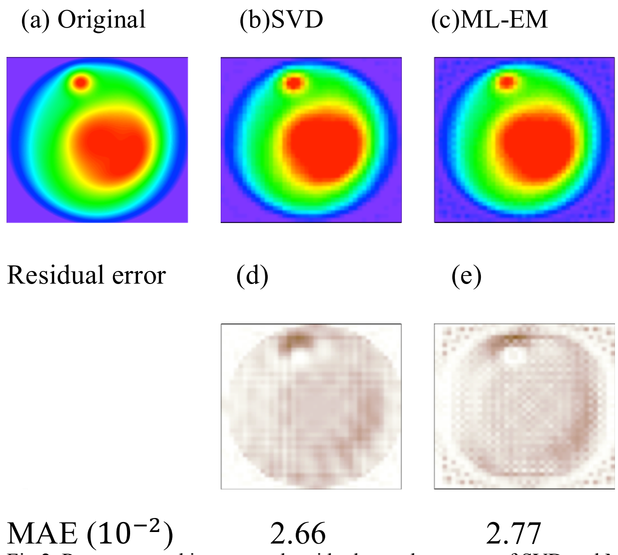

Fig.2. Reconstructed images and residual error by means of SVD and MLEM (a) Original image, (b) reconstructed image by SVD, (c)reconstructed image by ML-EM, (d)residual error image for (b), and residual error for (c), respectively.

References

[1] N. Iwama Computed Tomography for Plasma Measurement (1992)

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