A New Method of Electron Density Measurement by Fabry-Perot Interferometry

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A new method for determining the electron density of a thin plasma by means of Fabry-Perot interferometry is proposed. The interferometer consists of two plasma layers and dielectric material surrounded by two plasma layers. The transmittance of electromagnetic waves across the interferometer is calculated, and Fabry-Perot resonances are demonstrated. It is shown that the plasma density can be determined based on the measurement of the resonance frequency when the width of a plasma layer is known.

KEYWORDS: Fabry-Perot resonance, interferometry, electromagnetic wave, density measurement, micro-plasma

A Fabry-Perot interferometer is often used for spectroscopic measurements of visible light[1-3] and x-ray[4]. We here propose a new method for determining the electron density of a thin plasma by means of Fabry-Perot interferometry. A Fabry-Perot interferometer using thin plasmas as the resonator is shown in Figure 1. We assume that uniform plasma is confined by a very thin material which is transparent for electromagnetic waves, and for the sake of simplicity, we here neglect the plasma-confining material. A dielectric material with the dielectric constant $\varepsilon_a$ is inserted between two thin-plasma layers. If the wave frequency $\omega$ is larger than the electron plasma frequency $\omega_{pe}$, the wave is in a propagating mode, and otherwise the wave becomes a evanescent mode. In this article, we can show that the Fabry-Perot resonance occurs and has very sharp peak in $\omega$ for wave-evanescent over-dense plasmas ($\omega < \omega_{pe}$), and thus the Fabry-Perot interferometer can attain its high resolution[5,6]. We can therefore determine the electron density of a thin plasma from the measurement of the resonance frequency because the resonance frequency is dependent on $\omega_{pe}$.

Our starting point is a one-dimensional Maxwell wave equation given by

$$\frac{d^2}{dz^2} + k^2 \varepsilon(\omega, z) E(z) = 0,$$

with

$$\varepsilon(\omega, z) = \begin{cases} 
1, & \quad z < 0 \\
1 - \left( \frac{\omega_{pe}^2}{\omega} \right)^2, & \quad 0 \leq z \leq L \\
\varepsilon_a, & \quad L < z < 2L \\
1 - \left( \frac{\omega_{pe}^2}{\omega} \right)^2, & \quad 2L \leq z \leq 3L \\
1, & \quad z > 3L
\end{cases}$$

where $k = \alpha L/c$, $c$ is the speed of light, $\omega_{pe} = (e^2 n_p/\varepsilon_0 m)^{1/2}$ is the electron plasma frequency with a constant density $n_p$, $m$ the electron mass, $e$ the electric charge, and $\varepsilon_0$ the permittivity of free space. The solution of eq.(1) with eq.(2) is given by, for over-dense plasmas ($\omega < \omega_{pe}$),

$$E = \begin{cases} 
E_0 e^{ikz} + b e^{-ikz}, & \quad z < 0 \\
c_1 e^{ikz} + d_1 e^{-ikz}, & \quad 0 \leq z \leq L \\
f e^{ikz} + g e^{-ikz}, & \quad L < z < 2L \\
c_2 e^{ikz} + d_2 e^{-ikz}, & \quad 2L \leq z \leq 3L \\
\alpha e^{ikz}, & \quad z > 3L
\end{cases}$$

with

$$\lambda = k\sqrt{\varepsilon_a} = k\sqrt{(\omega_{pe}/\omega)^2 - 1}$$

and

$$k_2 = k\sqrt{\varepsilon_a}$$

where the eight coefficients $a, b, c_1, c_2, d_1, d_2, f$ and $g$ are determined from the continuity conditions of $E$ and its derivative at $z = 0, L, 2L,$ and $3L$. After rather lengthy calculations, we can obtain the transmittance $T$ of electromagnetic waves traversing this Fabry-Perot interferometer. We note that the transmittance $T$ is a function of three parameters, that is, $\alpha, \omega_{pe}, \varepsilon_a$, and $\omega_{pe}L/c$.

Fig.1 Schematic of Fabry-Perot interferometer using plasma
We first show the wave transmittance $T$ as a function of $aωpe$ for $ε_s = 1$ (i.e., a vacuum) and three different values of $ωpeL/c (=1, 3$ and $5)$ in Fig.2. When $ωpeL/c = 1$, the transmittance $T$ monotonously decreases with the decrease of $aωpe$, and no Fabry-Perot resonances appear. However, Fabry-Perot resonances can arise for $ωpeL/c = 3$ and $5$. We have one resonance for $ωpeL/c = 3$ and two resonances for $ωpeL/c = 5$. We see that the number of the resonances increases with the increase of $ωpeL/c$, and the resonance peak becomes sharper for the larger value of $ωpeL/c$. In Figure 3, we show the Fabry-Perot resonance frequencies as a function of $ωpeL/c$ for $ε_s = 1$. The resonances up to the fifth resonance are shown in the figure. Each resonance frequency decreases with the increase of $ωpeL/c$.

![Fig.2 Transmittance T as a function of aωpe for ε_s = 1 and aωpeL/c = 1, 3, and 5.](image)

We next mention a method for determining the electron density of thin plasmas used in the Fabry-Perot interferometer. We concentrate on the first resonance frequency, which is detected primarily by upward frequency sweeping. The first resonance frequency $ωk$ shown in Fig.3 can be well fitted by exponential functions as, for $1 ≤ x ≤ 16$,

$$a_k(x) = a_{pe} \left[ a_0 + \sum_{i=1}^{3} a_i \exp(-x/l_i) \right]. \tag{5}$$

where $x = ωkL/c$. If the plasma thickness $L$ is known in eq.(5), we can determine the plasma density $n_p$ through $ωk$ by measuring the first resonance frequency $ωk$, because eq.(5) is a function of $ωpe$ and $ωk$. In Figure 4, we show the relationship between the plasma density $n_p$ and the first resonance frequency $ωk$ for $ε_s = 1$ and different values of $L$. Thus, we see that we can determine the electron density from the measurement of the first resonance frequency by Fabry-Perot interferometry. It is also found that the resonance frequency shifts to the lower frequency side for the fixed $L$ and $n_p$ when the dielectric constant $ε_s$ increases. Finally, we consider that the present method can be applied to the electron density measurement of micro-plasmas such as PDP plasmas and semiconductor plasmas. However, a more realistic model should be necessary for the electron density measurement of industrial PDP plasmas.

![Fig.3 Resonance frequencies ωk/ωpe as a function of ωpeL/c for ε_s =1.](image)

![Fig.4 Plasma density n_p as a function of ωpe [GHz] for ε_s = 1 and L = 1, 2 and 3mm.](image)

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