Stationary Solitary Electomagnetic Waves Generated in Relativistic Laser-Plasma Interaction

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(Received 25 April 2002/ Accepted 27 May 2002)

Stationary solitary electromagnetic waves generated in ultraintense relativistic laser-plasma interactions are studied numerically. For one-dimensional stationary propagation with circular polarization, subcycle solitary waves moving with finite velocity are obtained for a given value of wave velocity and nonlinear frequency shift, and it is shown that the solitary-wave amplitude increases for the larger nonlinear frequency shift. A new type of solitary-wave solution, a paired solution of positive and negative solitary waves, is also shown.

Keywords: solitary wave, subcycle wave, ultraintense field, laser-plasma interaction, relativistic plasma

In ultraintense relativistic laser interactions with plasmas[1-8], Eriskepov *et al.*[1] have found an exact solitary-wave solution for the relativistic electromagnetic wave in one-dimensional stationary propagation. Its solution is a standing wave and is called a postsoliton[8]. Hojo and Akimoto[2] have found numerically that the relativistic wave equations derived by Eriskepov *et al.* have also a soliary-wave solution moving with finite velocity.

The aim of this paper is to study the features of the soliary-wave solution with finite moving velocity obeying the relativistic wave equations discussed by Eriskepov *et al.*[1], and also to show a new type of solitary-wave solution, that is, a paired solution in which positive and negative electromagnetic solitary waves are combined.

The starting point is Maxwell equations, and the relativistic fluid equations of the electron in one dimension given by

$$\frac{\partial}{\partial t} n + \frac{\partial}{\partial z}(n v_z) = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z}\right)\boldsymbol{P} = -e(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}) , \tag{2}$$

$$\frac{\partial}{\partial t}\boldsymbol{B} = -\nabla\times\boldsymbol{E} , \tag{3}$$

$$\nabla\times\boldsymbol{B} = \frac{1}{c^2}\frac{\partial}{\partial t}\boldsymbol{E} + \mu_0\boldsymbol{J} \tag{4}$$

with

$$\boldsymbol{P} = m\gamma\boldsymbol{v}, \quad \boldsymbol{J} = -en\boldsymbol{v}, \quad \gamma = (1 - v^2/c^2)^{-1/2},$$

where n is the electron density, $\boldsymbol{v}$ the electron flow velocity, $\boldsymbol{P}$ is the momentum, $\boldsymbol{J}$ the plasma current, and $\boldsymbol{E}$ and $\boldsymbol{B}$ the electric and magnetic wave fields, respectively. The ion is assumed to be immobile. If we introduce vector and scalar potentials, $\boldsymbol{a}$ and ϕ, defined respectively by

$$\boldsymbol{E} = -\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{a} , \qquad \boldsymbol{B} = \nabla\times\boldsymbol{a} \tag{5}$$

with $\nabla\cdot\boldsymbol{a} = 0$, considering a circularly polarized mode with $a_x + \mathrm{i}a_y = a(\xi)\exp\{i\omega[(1-V^2)t - V\xi]\}$ in one-dimensional stationary propagation, we can obtain[1]

$$\frac{\mathrm{d}^2}{\mathrm{d}\xi^2} a = \frac{1}{1-V^2}\frac{a}{\gamma - b} - \omega^2 a, \tag{6}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}\xi^2}\phi = \frac{b}{\gamma - b} , \tag{7}$$

$$\gamma = \sqrt{1 + a^2 + V^2 b^2} = 1 + \phi + V^2 b, \tag{8}$$

where $\xi = z - Vt$ and V is the velocity of a stationary solitary wave normalized by the velocity of light c. The following normalization was used: $\omega_{\mathrm{pe}} t \to t$, $\omega_{\mathrm{pe}} z/c \to z$, $\boldsymbol{P}/mc \to \boldsymbol{P}$, $e\boldsymbol{a}/mc \to \boldsymbol{a}$, $e\phi/mc^2 \to \phi$, $n/n_0 \to n$, where $\omega_{\mathrm{pe}} = (e^2 n_0/\varepsilon_0 m)^{1/2}$ and n_0 is uniform density. Here, the normalized nonlinear frequency shift is given by $1-\omega$, because ω (normalized by ω_{pe}) comes up to 1 when $a \to 0$. For $V = 0$, eqs.(6)-(8) are reduced to

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$$\frac{d^2}{d\xi^2}a = a\left[\gamma - \omega^2\gamma^2 + \frac{1}{\gamma^2}\left(\frac{da}{d\xi}\right)^2\right] \qquad (9)$$

with $\gamma = (1 + a^2)^{1/2}$ [2]. Equation (9) does not reduce to the stationary form of the nonlinear Schrödinger equation with cubic nonlinearity even to the third order in a due to the existence of of $(da/d\xi)^2$. However, it is shown that its exact solitary-wave solution[1] is reduced to the waveform of sech(ξ) in the lowest order of a.

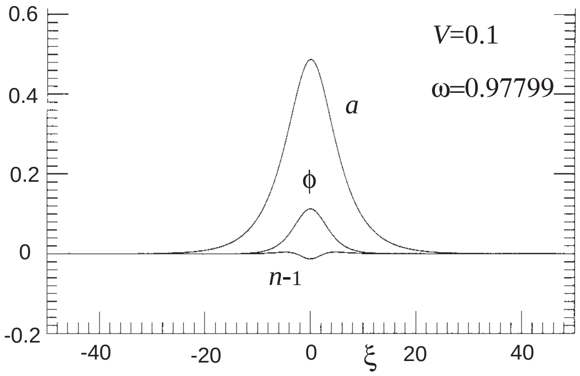

Fig.1(a) Solitary wave for V = 0.1 and ω = 0.97799.

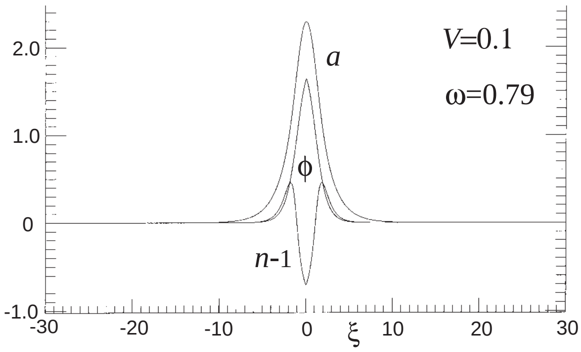

Fig.1(b) Solitary wave for V = 0.1 and ω = 0.79.

We now solve eqs.(6)-(8) numerically with the boundary condition in which the vector potential a and scalar potential ϕ vanish to zero at $|\xi| \to \infty$. In Fig.1, we show the solitary-wave solutions for V = 0.1 and ω = 0.97799 (a), 0.79 (b). The solitary-wave amplitude increases and the pulse-width decreases for the larger nonlinear frequency shift (i.e., for the smaller ω). We see that the profiles of ϕ and n become acute at $\xi \approx 0$ for a large-amplitude wave. Such a feature of the solution is also seen in the solutions obtained in Ref.[4]. In Fig.2, we show the maximum amplitudes of a and ϕ, and the minimum value of n as a function of ω. We can similarly obtain the solitary-wave solutions for different velocities, and the details will be reported elsewhere.

Finally, we show a new type of solitary-wave solution, i.e., a paired solution in which the positive and negative solitary waves are combined for V = 0.1 and ω = 0.90204 as shown in Fig.3. The waveform of a is a smooth function of ξ; however, a small ripple exists between two peaks in ϕ being coupled with the density oscillation, as seen in Fig.3.

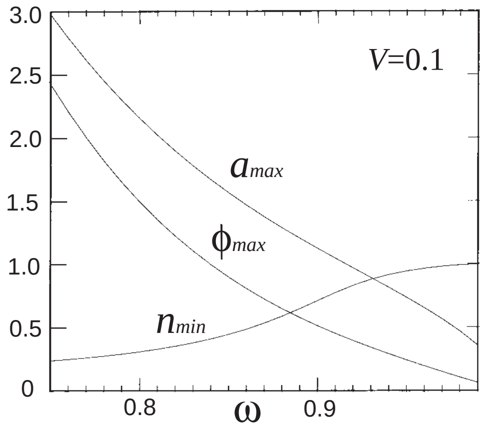

Fig.2 The wave amplitudes a_{max}, ϕ_{max} and n_{min} vs. ω for V = 0.1.

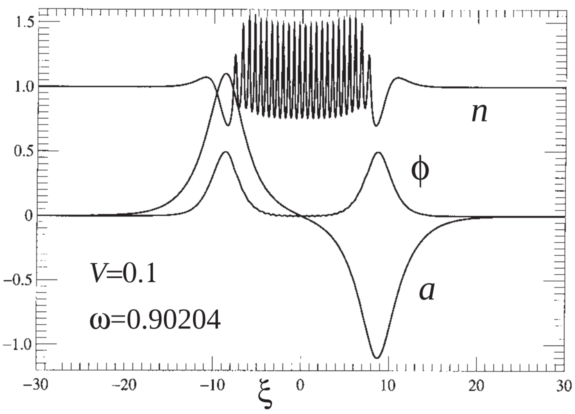

Fig.3 Paired solution of positive and negative pulses.

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