

# Full-Wave Maxwell Simulation on R-Mode Tunneling in Electron Cyclotron Resonance Heating Study

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A new numerical scheme for electromagnetic wave tracing is presented in place of the standard ray-tracing method in studies of electron cyclotron resonance heating. The new method solves the full-wave Maxwell equation, and can take into account wave diffraction, mode conversion (or, cross-polarization scattering), and wave tunneling across an evanescent region between resonance and cutoff layers, in addition to estimating energy absorption due to wave-particle resonances. One and two-dimensional simulations of electromagnetic wave tunneling are also demonstrated.

**Keywords:** electron cyclotron resonance heating, wave tracing, wave tunneling, wave absorption, Maxwell simulation

The standard ray-tracing method[1] used in electron cyclotron resonance heating analyses is based on geometrical optics, and cannot treat wave diffraction, mode conversion, and wave tunneling across an evanescent region between resonance and cutoff. On the other hand, Maxwell equations coupled with fluid equations can overcome the above difficulty associated with the standard ray-tracing method. However, wave-particle resonances such as cyclotron resonance cannot be taken into account in the Maxwell and fluid equations.

In this paper, we derive Maxwell and fluid equations in which fundamental cyclotron and Landau resonances are taken into account approximately to study electron cyclotron resonance heating problems. The starting point is the Maxwell wave equation given by

$$\nabla \times \nabla \times \mathbf{E} - \left( \frac{\omega}{c} \right)^2 \left[ 1 + i \frac{\vec{\sigma}}{\varepsilon_0 \omega} \right] \mathbf{E} = 0, \quad (1)$$

where  $\mathbf{E}$  is the electric wave field,  $\omega$  wave frequency,  $c$  the light speed,  $\varepsilon_0$  the permittivity of a vacuum,  $\vec{\sigma}$  the conductivity tensor. In the limit of zero Larmor radius[1], the conductivity tensor is given by,

$$i \frac{\vec{\sigma}}{\varepsilon_0 \omega} = \begin{pmatrix} S-1 & -iD & 0 \\ iD & S-1 & 0 \\ 0 & 0 & P-1 \end{pmatrix}, \quad (2)$$

$$S = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} + i \frac{\sigma_1}{\varepsilon_0 \omega},$$

$$D = \frac{\omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} + i \frac{\sigma_1}{\varepsilon_0 \omega},$$

$$P = 1 - \frac{\omega_{pe}^2}{\omega^2} + i \frac{\sigma_3}{\varepsilon_0 \omega},$$

$$\frac{\sigma_1}{\varepsilon_0 \omega} = \sqrt{\pi} \frac{\omega_{pe}^2}{2\omega |k_{\parallel}| v_{te}} \exp \left[ - \left( \frac{\omega - \omega_{ce}}{|k_{\parallel}| v_{te}} \right)^2 \right],$$

$$\frac{\sigma_3}{\varepsilon_0 \omega} = \sqrt{\pi} \frac{\omega \omega_{pe}^2}{(|k_{\parallel}| v_{te})^3} \exp \left[ - \left( \frac{\omega}{|k_{\parallel}| v_{te}} \right)^2 \right],$$

where  $\omega_{pe}$  is the electron plasma frequency,  $\omega_{ce}$  the electron cyclotron frequency,  $v_{te}$  the electron thermal velocity, and  $k_{\parallel}$  the parallel wavenumber. Here, we assume an infinite ion mass in obtaining eq.(2), which is valid in the frequency regime much higher than the ion cyclotron frequency. If the imaginary parts in  $S$ ,  $D$ , and  $P$  which show wave-particle resonances are neglected, the wave equation (1) with (2) is shown to be identical to the following equations[2,3]:

$$\frac{\partial}{\partial t} \mathbf{B} = -\nabla \times \mathbf{E}, \quad (3)$$

$$\frac{\partial}{\partial t} \mathbf{E} = c^2 \nabla \times \mathbf{B} - \frac{1}{\varepsilon_0} \mathbf{J}, \quad (4)$$

$$\frac{1}{\varepsilon_0} \frac{\partial}{\partial t} \mathbf{J} = \omega_{pe}^2 \mathbf{E} - \frac{e}{\varepsilon_0 m_e} \mathbf{J} \times \mathbf{B}_0, \quad (5)$$

where  $\mathbf{B}_0$  is an external magnetic field. Since eqs.(3)-(5) are time-evolution equations, we can trace the temporal behaviors of wave beams by solving them as an initial value problem. That is, we can use eqs.(3)-(5) for wave tracing in place of the ray-tracing equation. However, as they do not include wave-particle interactions, they cannot be used for estimating power deposition in electron cyclotron resonance heating

analysis. In order to solve this problem, we consider finding the time-evolution equations equivalent to eq.(1) with eq.(2) including wave-particle resonances. If we suppose that the imaginary parts of  $S$ ,  $D$ , and  $P$  are constants in real space even though they are actually functions of  $\omega$  and  $k_{\parallel}$  obtained in Fourier space, we can take into account the imaginary parts in  $S$ ,  $D$ , and  $P$  by replacing eq.(4) with the following equation:

$$\frac{\partial}{\partial t} \mathbf{E} = c^2 \nabla \times \mathbf{B} - \frac{1}{\epsilon_0} \mathbf{J} - \frac{1}{\epsilon_0} \vec{\sigma}_R \cdot \mathbf{E}, \quad (6)$$

$$\vec{\sigma}_R = \begin{pmatrix} \sigma_1 b_z^2 + \sigma_3 b_x^2 & -i\sigma_1 b_z & -(\sigma_1 - \sigma_3) b_x b_z \\ i\sigma_1 b_z & \sigma_1 & -i\sigma_1 b_x \\ -(\sigma_1 - \sigma_3) b_x b_z & i\sigma_1 b_x & \sigma_1 b_x^2 + \sigma_3 b_z^2 \end{pmatrix}, \quad (7)$$

where  $\mathbf{b} = \mathbf{B}_0 / B_0 = b_x \hat{x} + b_z \hat{z}$  is assumed as a 2-d model of an external magnetic field. Here, we also assume that  $k_{\parallel}$  in  $S$ ,  $D$ , and  $P$  is determined based on the local dispersion relation  $\epsilon(k_{\parallel}, \omega, x, z) = 0$  for right-handed circularly polarized modes (R-modes) in electron cyclotron resonance heating analysis.

In this case, the energy deposition into plasma due to wave-particle resonances is calculated from

$$\begin{aligned} W &= \frac{1}{2} \text{Re} \{ \mathbf{E}^* \cdot \vec{\sigma} \cdot \mathbf{E} \} \\ &= \frac{1}{2} \sigma_1 (b_z^2 |E_x|^2 + |E_y|^2 + b_x^2 |E_z|^2) \\ &\quad + \frac{1}{2} \sigma_3 (b_x^2 |E_x|^2 + b_z^2 |E_z|^2) \\ &\quad + \frac{1}{2} (\sigma_3 - \sigma_1) b_x b_z (E_x^* E_z + E_x E_z^*) \\ &\quad + \frac{i}{2} \sigma_1 [(b_z E_x - b_x E_z) E_y^* - (b_z E_x^* - b_x E_z^*) E_y], \end{aligned} \quad (8)$$

which can be reduced to

$$W = \frac{1}{2} \sigma_3 |E_z|^2 + \frac{1}{2} \sigma_1 |E_x - iE_y|^2 \quad (9)$$

when  $b_z = 1$  and  $b_x = 0$ . We see that the first and second terms in eq.(9) denote the energy depositions due to Landau and cyclotron resonances, respectively.

Hereafter, we show the simulation results regarding R-mode tunneling in electron cyclotron resonance heating study using eqs.(3), (5), and (6). We consider a simulation area being square in  $(x, z)$ . The R-mode which has a Gaussian profile in the  $z$ -direction is excited on the boundary of  $x = 0$ , and is launched in the positive  $x$  direction. We impose an out-going wave condition

for other three boundaries. In Fig.1, we show the temporal behaviors of R-mode ( $E_y$  evaluated at the beam-center in the 2-d simulation) traversing an evanescent region between resonance and cutoff layers, where are located near  $x = 1600$ . We assumed  $\omega = 28\text{GHz}$ ,  $T_e = 100\text{eV}$ ,  $n_0 = 10^{12}\text{cm}^{-3}$ , where a constant density and magnetic field profile of a tanh-type are assumed. In Fig.2, we show the transmittance  $T$  of R-mode traversing an evanescent region between the resonance and cutoff layers obtained in 1(●) and 2(■)-dimensional simulations, where  $L$  is the distance from the resonance to the cutoff layers, and  $k$  is defined by  $k = k(x=z=0)$ . Here, the transmittance is defined by  $T = S_x(x > \text{cutoff-layer}) / S_x(x < \text{resonance-layer})$ , where  $S_x$  is the x-component of the Poynting vector and is evaluated at the beam-center in the 2-d simulation. The solid line expresses  $T = \exp(-\pi k L)$ , which is the theoretical expectation obtained in the Budden's problem[1]. We see that the numerical results are in close agreement with the Budden's expectation.

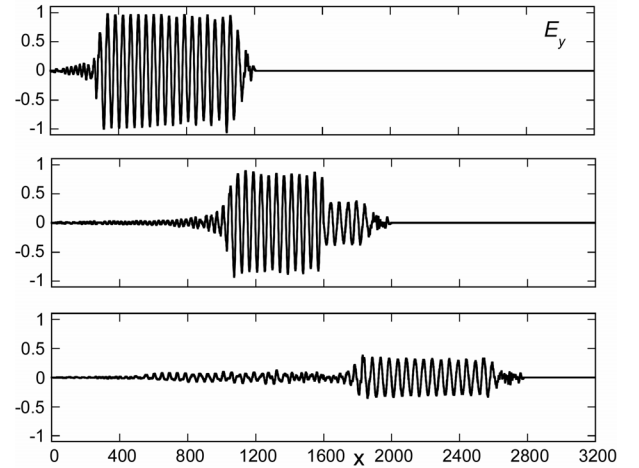


Fig.1 Wave tunneling of R-mode traversing an evanescent region near  $x = 1600$  in the 2-d simulation.

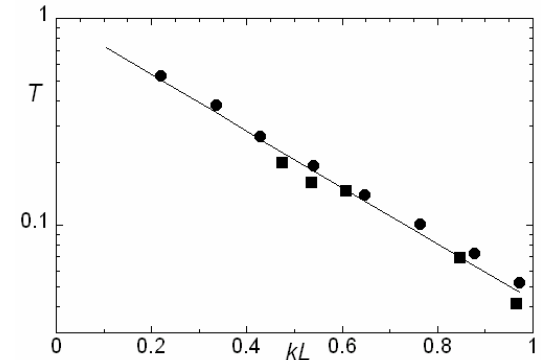


Fig.2 Transmittance  $T$  as a function of  $kL$ .

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