

Reflectionless Transmission of Electromagnetic Wave in One-Dimensional Multi-Layer Plasmas

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The reflectionless transmission of electromagnetic waves in one-dimensional multi-layer plasmas is studied. The wave transmittance is obtained analytically for single-layer underdense plasma as well as for two-layer critical plasma where the wave frequency ω is equal to the electron plasma frequency ω_{pe} , and it is shown that reflectionless transmission can be possible for both cases. Reflectionless transmission in two-layer critical plasma as well as in single-layer underdense plasma should be considered Fabry-Perot resonance well-known in optics.

Keywords: electromagnetic wave, transmittance, reflection transmission, multi-layer plasma, Fabry-Perot resonance

The reflection and transmission of electromagnetic waves in plasma layers is a basic problem in plasma physics, and its solution such as reflectionless transmission is of particular significance in regard to plasma's technological applications.

Here, we study electromagnetic-wave transmission in one-dimensional multi-layer plasma. Electromagnetic waves which are launched into a plasma layer are generally reflected from the plasma; however, it is well-known that waves can be perfectly transmitted without receiving reflections if a certain condition is satisfied. This is called the Fabry-Perot resonance[1]. This phenomenon can be realized for an underdense plasma layer, that is, $\omega > \omega_{pe}$. For a critical density or overdense plasma layer satisfying $\omega \leq \omega_{pe}$, reflectionless transmission does not occur since the waves are not propagating. However, we can show that reflectionless transmission can be possible for multi-layer plasmas even if $\omega = \omega_{pe}$. This should be considered Fabry-Perot resonance.

Our starting point is a one-dimensional Maxwell wave equation given by

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial z^2} + \omega_{pe}^2 \right) E(z, t) = 0, \quad (1)$$

where c is the speed of light, $\omega_{pe} = (e^2 n_p / \epsilon_0 m)^{1/2}$, and n_p is a plasma density. For the stationary wave propagation, assuming $E(t) \propto \exp(i\omega t)$, we obtain

$$\left(\frac{d^2}{dz^2} + \frac{\omega^2 - \omega_{pe}^2}{c^2} \right) E(z) = 0. \quad (2)$$

Here, we study electromagnetic-wave transmission based on eq.(2) for the multi-layer plasma shown in Fig.1. We first consider the case of a single-layer plasma ($n = 1$). We assume the plasma density $n(z)$ given by

$$n_p(z) = \begin{cases} 0, & z < 0 \\ n_0, & 0 \leq z \leq L \\ 0, & z > L. \end{cases} \quad (3)$$

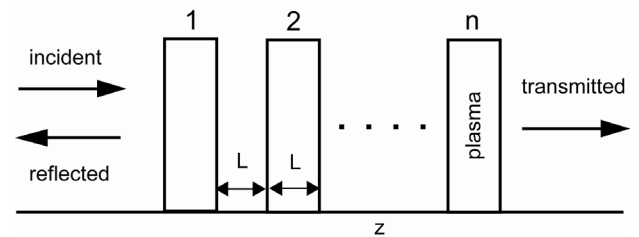


Fig.1 Schematic of multi-layer plasma

For $\omega > \omega_{pe}$, the solution of eq.(2) with eq.(3) is given by

$$E = \begin{cases} E_0 e^{ikz} + b e^{-ikz}, & z < 0 \\ c e^{ik_p z} + d e^{-ik_p z}, & 0 \leq z \leq L, \\ a e^{ikz}, & z > L \end{cases}, \quad (4)$$

where $k = \omega/c$, $k_p = (\omega^2 - \omega_{pe}^2)^{1/2}/c$, and E_0 is the

incident-wave amplitude. The four coefficients a , b , c , and d are determined from the continuity conditions of E and dE/dz at $z = 0$ and $z = L$. Substituting eq.(4) into the continuity conditions of E and dE/dz , we can obtain the wave transmittance $T (= |a/E_0|^2)$ given by

$$T = \frac{16\alpha^2}{(1+\alpha)^4 + (1-\alpha)^4 - 2(1-\alpha^2)^2 \cos(2kL\alpha)}, \quad (5)$$

where $\alpha = [1 - (\omega_{pe}/\omega)^2]^{1/2}$. We show the transmittance T as a function of $(\omega_{pe}/\omega)^2$ for $kL = 1, 6$, and 10 in Fig.2. We see that reflectionless transmission ($T = 1$) can occur for $kL = 6$ and 10 , which is well-known in optics as the Fabry-Perot resonance[1]. The number of the resonant frequency that corresponds to reflectionless transmission increases with the increase in the width of the plasma layer. For a critical-density plasma satisfying $\omega = \omega_{pe}$, taking a limit of $\alpha \rightarrow 0$, we obtain

$$T = \frac{1}{1 + \left(\frac{kL}{2}\right)^2}. \quad (6)$$

In this case, the transmittance T becomes a monotone decreasing function of kL . That is, the transmittance decreases with the increase of the plasma-layer width.

We next consider the wave transmission in the case of two-layer plasma ($n = 2$). For the sake of simplicity, we here assume a critical-density plasma with $\omega = \omega_{pe}$. In this case, the solution is given by

$$E = \begin{cases} E_0 e^{ikz} + b e^{-ikz}, & z < 0 \\ c_1 z + d_1, & 0 \leq z \leq L \\ f e^{ik_r z} + g e^{-ik_r z}, & L < z < 2L \\ c_2 z + d_2, & 2L \leq z \leq 3L \\ a e^{ikz}, & z > 3L \end{cases}, \quad (7)$$

where the coefficients a , b , c_1 , c_2 , d_1 , d_2 , f , and g are determined from the continuity conditions of E and its derivative at $z = 0, L, 2L$, and $3L$. By means of a moderately lengthy calculation, we obtain the wave transmittance in the case of two-layer plasma as

$$T = \frac{\mu^4}{1 + (1 + \mu^2)^2 - 4\mu \sin\left(\frac{4}{\mu}\right) - 2(1 - \mu^2) \cos\left(\frac{4}{\mu}\right)}, \quad (8)$$

with $\mu = 2/kL$. In this case, the transmittance becomes a function of kL only. In Fig.3, we show the transmittance T as a function of kL . We see that in this

case as well, the reflectionless transmission of electromagnetic waves can be possible as shown in Fig.3. This, as in the case of single-layer underdense plasma ($\omega > \omega_{pe}$), should be considered the Fabry-Perot resonance.

We note that the reflectionless transmission of electromagnetic waves due to the Fabry-Perot resonance as shown in Fig.3 can arise for over-dense plasmas with $\omega < \omega_{pe}$ [2], and is also possible for plasmas of diffusive profiles from the analogy with the reflectionless potential scattering discussed in quantum mechanics[3]. A similar method for measuring the electron density of sheet plasmas is proposed in Ref.4, though this method is not concerned with the Fabry-Perot resonance. Such reflectionless transmission due to the Fabry-Perot resonance in multi-layer plasmas can be applied to frequency filters[5,6] and interferometers[7,8] in the micro and millimeter-wave range, and also might be applied to measurements of the electron density of plasma display panel(PDP) plasmas.

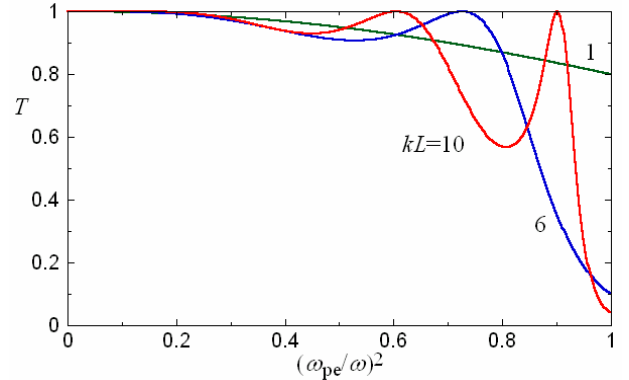


Fig.2 Transmittance T as a function of $(\omega_{pe}/\omega)^2$ for single-layer plasma, where $kL = 1, 6$ and 10 ($k = \omega/c$).

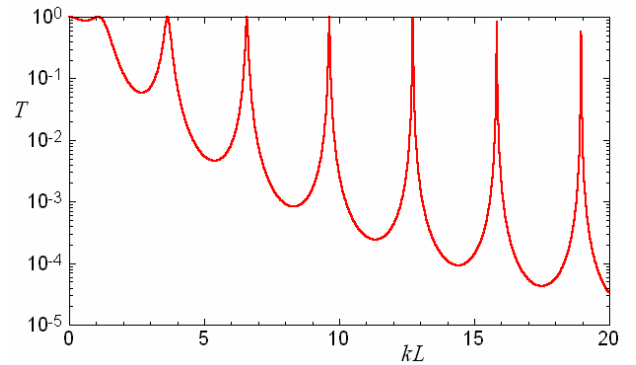


Fig.3 Transmittance T as a function of kL for two-layer plasma with $\omega = \omega_{pe}$ ($k = \omega/c$).

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- [1] M. Born and E. Wolf, *Principles of Optics* (Pergamon Press, New York, 1970).
- [2] H. Hojo and A. Mase, Extended Abstracts in Optics Japan 2003 (Hamamatsu, 2003), p.230.
- [3] L.D.Landau and E.M.Lifshitz, *Quantum mechanics, non-relativistic theory* (Pergamon Press, 1958).
- [4] A. Ejiri and K. Kawahata, Jpn. J. App. Phys. **39**, 2839 (2000).
- [5] M. Scalora *et al.*, J. Appl. Phys. **83**, 2377 (1998).
- [6] C. Pacher *et al.*, Appl. Phys. Lett. **79**, 1486 (2001).
- [7] R.L.Waters and M.E.Aklufi, Appl. Phys. Lett. **81**, 3320 (2002).
- [8] Yu.V. Shvyd'ko *et al.*, Phys. Rev. Lett. **90**, 013904 (2003).