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Discussion on Entropy and Temperature of a Non-Equilibrium Plasma with Low Density based on Statistical Physics

低密度非平衡プラズマの温度に関する統計力学的考察

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In this study, we reconsider electron temperature of non-equilibrium plasmas based on thermodynamic and statistical-mechanics physics by rigorous calculation of their entropy. As a concrete procedure, we carry out the following things. In the first place, we solve Boltzmann equation, and we obtain EEDF of the non-equilibrium plasma. In the second place, by the method of the statistical-mechanics, we obtain the mean electronic energy interpreted as internal energy and entropy from EEDF and describe entropy for a function of the energy. Furthermore, based on thermodynamic principle, we obtain electron "temperature" which have non-equilibrium EEDF by the taking of derivative of entropy as we discussed previously, with respect to mean electron energy, and taking its reciprocal number. Finally, we discuss about "Temperature" above from statistical thermodynamics-like viewpoint.

1. Introduction

Generally, in a plasma with low enough ionization degree, the electron energy distribution function (EEDF) is far from a Maxwellian one. This is due to mainly elastic and inelastic electron collision processes with the atoms or molecules. These collision processes are more dominant than a coulomb collision processes in a weakly ionized plasma. In addition, electron collisions in plasmas contribute essential processes in atoms and molecules, such as electron-impact excitation, dissociation and ionization processes. As a result, since a high energy electron is consumed, the EEDF becomes depleted one in high enegy region.

In this study, we assumed EEDF which have electron population depleted in high energy region compared with maxwellian one, to calculate the electron "temperature" by means of statistical thermodynamic procedure.

2. Calculational Model

We now consider the EEDF which has electron population depleted in high energy region compared with the Maxwellian one.

2.1 *Electron velocity distribution function*

In the present study, we approximate the EEDF to be a non-equilibrium one which is constructed in two-temperature region as follows:

$$\begin{cases} f_{T_1}(v_x) = A\exp\left(-\dfrac{m}{2k_BT_1}v_x^2\right) \ , & 0 \le |v_x| \le v_{x0} \\ f_{T_2}(v_x) = B\exp\left(-\dfrac{m}{2k_BT_2}v_x^2\right) \ , & |v_x| > v_{x0} \end{cases} \qquad (1)$$

where A and B are normalization factor, m is electron mass, k_B is the Boltzmann constant, v_{x0} is electron population velocity which has threshold energy E_0. T_1 is temperature of electron population in low energy region, T_2 is temperature of electron population in high energy region, and their values also correspond to a slope of function in which we take the logarithm of both side for Eq. (1). Figure 1 presents the general form of EEDF for the equation of (1) as a semi-log plot.

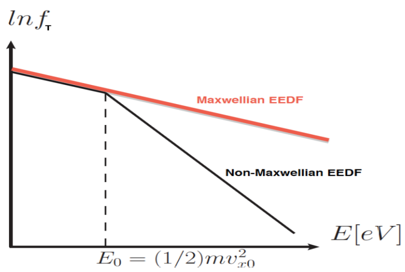

Fig.1. General form of EEDF.

The normalization factor A and B are given by the continuity condition $f_{T1}(v_{x0}) = f_{T2}(v_{x0})$ and the integral of f_T over all electron velocities.

$$\int_{-\infty}^{+\infty} f(v_x)dv_x = 1 \qquad (2)$$

Solving the integral, we obtain

$$\begin{cases} A = \sqrt{\dfrac{m}{2\pi k_B}}\dfrac{\exp\left(\frac{E_0}{k_BT_1}\right)}{\sqrt{T_1}\mathrm{erf}\left(\sqrt{\frac{E_0}{k_BT_1}}\right)\exp\left(\frac{E_0}{k_BT_1}\right)+\sqrt{T_2}\mathrm{erfc}\left(\sqrt{\frac{E_0}{k_BT_2}}\right)\exp\left(\frac{E_0}{k_BT_2}\right)} \\ B = \sqrt{\dfrac{m}{2\pi k_B}}\dfrac{\exp\left(\frac{E_0}{k_BT_2}\right)}{\sqrt{T_1}\mathrm{erf}\left(\sqrt{\frac{E_0}{k_BT_1}}\right)\exp\left(\frac{E_0}{k_BT_1}\right)+\sqrt{T_2}\mathrm{erfc}\left(\sqrt{\frac{E_0}{k_BT_2}}\right)\exp\left(\frac{E_0}{k_BT_2}\right)} \end{cases} \qquad (3)$$

2.2 *Mean electron energy*

The mean electron energy is given by the following equation.

$$<E>=\left\langle \frac{1}{2}mv_x^2 \right\rangle + \left\langle \frac{1}{2}mv_y^2 \right\rangle + \left\langle \frac{1}{2}mv_z^2 \right\rangle \qquad (4)$$

where we have assumed that the electron distribution in plasma is isotropic, noting that $<\frac{1}{2}mv_x^2>=<\frac{1}{2}mv_y^2>=<\frac{1}{2}mv_z^2>$. Hence, the mean electron energy of EEDF is given by the integral of $f_T(v)$ over all velocity as follows:

$$\left\langle \frac{1}{2}mv_x^2 \right\rangle = \frac{1}{2}m \int_{-\infty}^{\infty} v_x^2 f(v_x) dv_x \qquad (5)$$

2.3 *Entropy*

According to *Shannon entropy,* we can calculate entropy by means of distribution function $f_T(v)$ defined as Eq. (1), noting that $f_T(v_x) = f_T(v_y) = f_T(v_z)$.

$$S = -k_B \iiint_{-\infty}^{+\infty} f(v_x)f(v_y)f(v_z) \ln[\, f(v_x)f(v_y)f(v_z)\,]\, dv_x dv_y dv_z \qquad (6)$$

2.4 *Temperature*

Concerning electron "temperature" which have non-equilibrium EEDF, we calculate the partial derivative of entropy with respect to mean electron energy in plasmas from a thermodynamic relationship as follows:

$$\frac{\partial S}{\partial <E>} = \frac{1}{T} \qquad (7)$$

2.4 *Input parameter*

Figure 2 presents the numerical result of EEDF in oxygen plasma with respect to reduced electric field. In this figure, there is a logarithmic plot of EEDF which shows Maxwellian distribution as straight red lines. We can see threshold energy point at $E_0 = 6.75$ eV in Fig 2. In low energy region, EEDF can approximate to a Maxwellian EEDF until the threshold energy point.

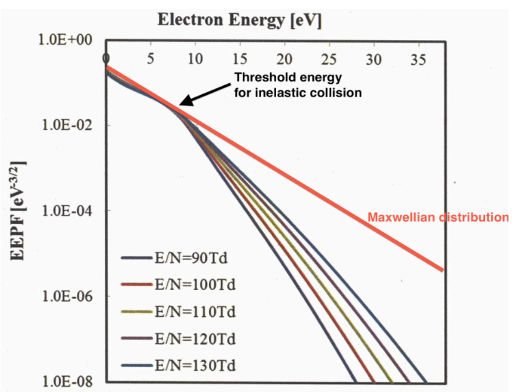

Fig. 2. Numerical result of EEDF with respect to reduced electric field.

Its slope corresponds to value of temperature T_1. Also value of temperature T_2 with respect to reduced electric field *E/N* corresponds to slope of other lines, such as blue and red, yellow, purple, indigo blue one.

3. Results and Discussion

We studied electron temperature of non-equilibrium oxygen plasmas based on thermodynamic and statistical-mechanical physics by rigorous calculation of their entropy which obtained from information of general form of EEDF.

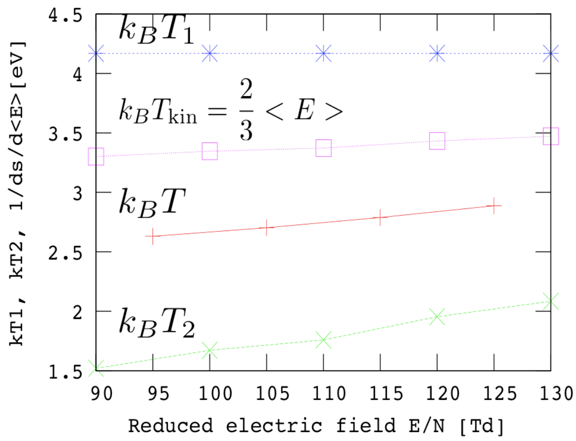

Fig.3. Various temperature.

In Fig. 3, we compare the electron temperature that we calculated by mean electron energy which shows pink lines to the electron "Temperature" that we calculated by entropy which shows red lines. In the non-equilibrium plasma with low density, generally, the temperature range from 0 to 5 eV. Hence, in this study, we think that it is not irrelevant approach that we get to the bottom of a temperature of the plasma which obtain from a EEDF.

References

[1] R. Alvarez, J. Cotrino and A. Palmero: EPL. **105** (2014) 15001.

[2] R. Hippler, H. Kersten, M. Schmid, K. H. Schoenbach, *Low Temperature Plasma*, second, Revised and Enlarged Edition, WILEY-VCH Verlag GmbH & Co KGaA, Weinheim, (2008).

[3] Y. Shimizu, A. Nezu, H. Matsuura, H. Akatsuka: IEE Japan. PST. **12**(2012) 004.