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Reduction in the strong singularity encountered in the reconstruction analysis of eddy current profile

渦電流分布逆解析で現れる強い特異性の除去

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In a reversed field pinch device, the ordinary Cauchy-condition surface (CCS) method cannot reconstruct the magnetic flux profile accurately due to the strong eddy current flow on the shell. Boundary integrals of the unknown eddy current density along the shell are added to the original CCS method formulation. As the sensors are closely adjacent to the shell, the singular boundary integrals should be accurately evaluated. This singularity can be damped out with the present subtraction technique based on the distance function algorithm. The capability of the method is demonstrated for a test problem modelling the RELAX device.

1. Introduction

As strong eddy currents exist on the shell of the reversed field pinch device, RELAX, the Cauchy condition surface (CCS) method [1] based on magnetic sensor signals needs to be modified to reconstruct accurately the magnetic flux profile in the device. The new term

$$\mu_0 \int_{\Gamma_{\text{Shell}}} j_S(\mathbf{r}_l) B^*(\mathbf{r}_l \to \mathbf{r}_i) d\Gamma(\mathbf{r}_l) \qquad (1)$$

is added to each boundary integral equation in the CCS method formulation. Here B^* is the fundamental solution ψ^* or its derivative, while j_S denotes the linear density distribution of the eddy current on the shell, which is integrated in the poloidal direction along the shell.

2. Method

The boundary integral along the shell, Eq.(1), should be performed very carefully. Since the distance $\varepsilon = \sqrt{(r-a)^2 + (z-b)^2}$ between the sensor position (*a*,*b*) and an integration point (r, z) on the shell is very short, the following singularities [2]

$$\psi^* \to -\frac{a}{2\pi} \log \varepsilon , \qquad (2)$$

$$\frac{\partial \psi^*}{\partial a} \to -\frac{1}{4\pi} \log \varepsilon + \frac{a(r-a)}{2\pi} \cdot \frac{1}{\varepsilon^2} \qquad (3)$$

and

$$\frac{\partial \psi^*}{\partial b} \to \frac{a(z-b)}{2\pi} \cdot \frac{1}{\varepsilon^2} , \qquad (4)$$

arise in the integration kernels when $\varepsilon \to 0$.

In a boundary element ($\Gamma_{\text{Shell},j}$) along the shell, one here uses the notations $G(\xi)$, $\phi(\xi)$ and $F_S(\xi)$ for the Jacobian, the interpolation function, and the corresponding asymptotic function (Eqs.(2), (3) or (4)), respectively. The general form of the boundary integral over $\Gamma_{\text{Shell},j}$ is rearranged as

$$\int_{-1}^{1} \phi(\xi) G(\xi) B^*(\xi) d\xi = \int_{-1}^{1} \left\{ \phi(\xi) G(\xi) B^*(\xi) - \phi_0 G_0 F_S(\xi) \right\} d\xi + \phi_0 G_0 \int_{-1}^{1} F_S(\xi) d\xi, \qquad (5)$$

where G_0 and ϕ_0 are the values of $G(\xi)$ and $\phi(\xi)$ at $\xi = \xi_0$ on the boundary element that is the nearest to the sensor under consideration. In Eq.(5), the asymptotic function on the RHS is subtracted from the original integrand, and this subtraction is compensated by the analytical integral (the second integral). The total integrand of the first integral has no singularity and can be evaluated with the ordinary Gaussian quadrature.

Ma and Kamiya [3] proposed the use of a 'distance function' for the boundary element adjacent to the sensor position (*a*,*b*) as

$$d(\xi) = G_0 \sqrt{(\xi - \xi_0)^2 + (d_0 / G_0)^2} , \qquad (6)$$

where d_0 is the minimum distance from the point (*a*,*b*) to the boundary element as shown in Fig.1, which corresponds to the local coordinate $\xi = \xi_0$. Equation (6) agrees with ε in Eqs.(2), (3) and (4) when $\xi \to \xi_0$ and $d_0 \to 0$.

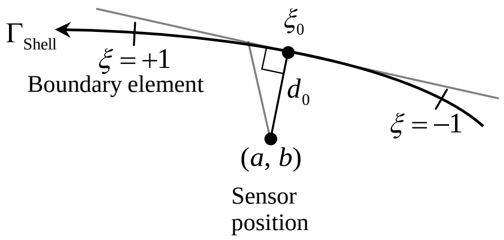


Fig.1. The minimum distance d_0

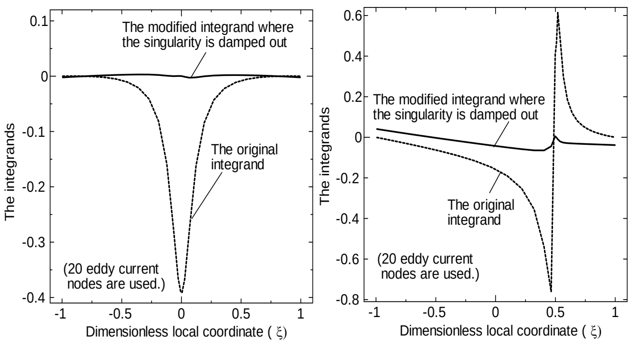
For instance, a quantity in Eq.(3) can be rewritten in the following form:

$$\frac{a(r-a)}{2\pi}\cdot\frac{1}{\varepsilon^2}=\frac{a}{2\pi G_0^2}$$
$$\times\left[\alpha_0+\frac{\alpha_1(\xi-\xi_0)}{(\xi-\xi_0)^2+(d_0/G_0)^2}+\frac{\alpha_2}{(\xi-\xi_0)^2+(d_0/G_0)^2}\right] \quad (7)$$

The terms having a singularity can be integrated analytically, e.g.,

$$\int_{-1}^{1}\frac{(\xi-\xi_0)}{(\xi-\xi_0)^2+(d_0/G_0)^2}d\xi$$
$$=\log\sqrt{(1-\xi_0)^2+\left(\frac{d_0}{G_0}\right)^2}-\log\sqrt{(1+\xi_0)^2+\left(\frac{d_0}{G_0}\right)^2}. \quad (8)$$

Figure 2(a) and 2(b) are examples to compare the integrands in terms of $\partial\psi^*/\partial b$ before and after damping out the singularity. The strong singularities are efficiently damped out.



(a) $\xi_0 = 0.0$, $d_0 = 8$mm (b) $\xi_0 = 0.5$, $d_0 = 2$mm

Fig.2. Integrands before and after damping out the singularity

3. Numerical Examples

One here deals with a limiter configuration of the RELAX device, assuming 40 flux loops and 40 tangential probes inside the shell. The CCS is placed in a domain that can be supposed to be inside the plasma. Figure 3 shows the variation in the eddy current density on the shell for the cases assuming 40 eddy current nodes. The vertical axis denotes the current density, while the abscissa means the poloidal angle. The dashed and the solid curves in Fig.3 denote the reference and the reconstructed variation in the eddy current density, respectively. The reference solution was obtained using the RELAX-Fit code [4].

Figure 4 shows the reconstructed flux profiles for the 40 current node case. The dashed contours show the reference solution, while the solid contours indicate the reconstructed flux. Accurate results can be observed even deep inside the plasma region.

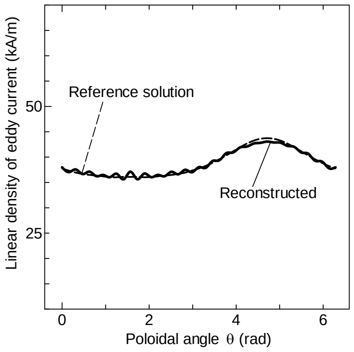


Fig.3. Reconstruction of the eddy current profile.

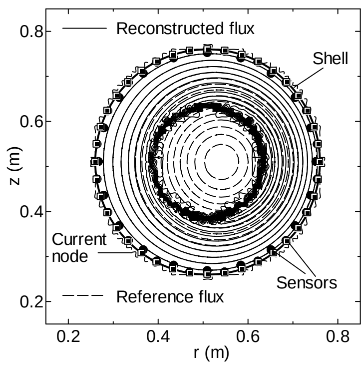


Fig.4. Reconstruction of the magnetic flux profile.

4. Conclusions

The singularity in the boundary integral along the shell can be damped out effectively by introducing the present subtraction technique with the distance function algorithm.

Acknowledgments

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