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Relaxation Process in Pure Electron Plasmas Revealed by Fokker-Plank-type Kinetic Equation

Fokker-Planck型衝突項を有する運動論的方程式による
純電子プラズマの緩和過程の理解

Yuichi Yatsuyanagi, Tadatsugu Hatori[1] and Yukihiro Soga[2]
八柳祐一, 羽鳥尹承, 曽我之泰

Faculty of Education, Shizuoka University, 836 Oya, Suruga-Ku, Shizuoka 422-8529, Japan
静岡大学教育学部　〒422-8529　静岡市駿河区大谷836
[1)] *National Institute for Fusion Science, 322-6 Oroshi-cho , Toki, Gifu 509-5292, Japan*
核融合科学研究所　〒509-5292　土岐市下石町322-6
[2)]*Institute of Science and Engineering, Kanazawa University,Kakuma-cho, Kanazawa, Ishikawa 920-1192, Japan*
金沢大学理工研究域　〒920-1192　金沢市角間町

To reveal the detailed relaxation process of the nonneutral plasma, H theorem is numerically examined using the previously obtained kinetic equation for the two-dimensional single-species point vortex system.

1. Introduction

Motion of electrons confined radially by an strong axial magnetic field and axially by end negative potentials in a vacuum chamber is determined by the inviscid two-dimensional (2D) Euler equation if the cyclotron motion of the electron can be ignored ($\boldsymbol{E}\times\boldsymbol{B}$ drift approximation). The two-dimensional motion perpendicular to the magnetic field of the electrons can be traceable numerically by the point vortex method. Thus, we focus on the point vortex model from now on.

In this paper, we will demonstrate the possibility of three-stage relaxation mechanism in the point vortex system, which consist of violent, fast-slow and slow-slow relaxations via H theorem.

2. Kinetic equation and H theorem

We have obtained a kinetic equation for a two-dimensional point vortex system. The point vortex system has a collisional effect due to its discrete, pointwise distribution of the vorticity in a macroscopic fluid scale. The obtained kinetic equation describes an evolution of the system in a macroscopic fluid scale. The collision term consists of a diffusion term and a drift term.

$$\frac{\partial}{\partial t}\omega+\nabla\cdot(\boldsymbol{u}\omega)=-\nabla\cdot\boldsymbol{\Gamma}$$
$$\Gamma=-\mathrm{D}\cdot\nabla\omega+\boldsymbol{V}\omega$$
$$\mathrm{D}=K\int d\boldsymbol{r}'\frac{(\boldsymbol{u}-\boldsymbol{u}')(\boldsymbol{u}-\boldsymbol{u}')}{\left|\boldsymbol{u}-\boldsymbol{u}'\right|^3}\omega'$$
$$\boldsymbol{V}=-K\int d\boldsymbol{r}'\frac{(\boldsymbol{u}-\boldsymbol{u}')(\boldsymbol{u}-\boldsymbol{u}')}{\left|\boldsymbol{u}-\boldsymbol{u}'\right|^3}\cdot\nabla'\omega'$$

where ω is vorticity and $\boldsymbol{u}$ is velocity. The notations D and $\boldsymbol{V}$ represent the diffusion tensor and drift velocity, respectively. The physical properties of the collision term for absolute negative temperature system is as follows: (a) the diffusion term decreases the system energy, while the drift term increases it. As a whole, macroscopic total system energy conserves, (b) the diffusion term increases the entropy, while the drift term decreases it. As a whole, the system entropy increases during the relaxation process and the entropy production rate becomes zero when a system reaches a global thermal equilibrium state.

Let us examine the entropy production (H theorem) in detail. The H function is defined by

$$\begin{aligned}H&=\int d\boldsymbol{r}\frac{\omega(\boldsymbol{r})}{\Omega}\ln\frac{\omega(\boldsymbol{r})}{\Omega}+\mathrm{const.}\\&=\int d\boldsymbol{r}\,n(\boldsymbol{r})\ln n(\boldsymbol{r})+\mathrm{const.}\end{aligned}\qquad(3)$$

We test the H theorem numerically. To elucidate the temporal evolution of the H function, a non-axisymmetric profile is chosen as an initial configuration. Typical time evolution of the point vortex system is shown in Fig. 1.

Relaxation time is defined by a decay constant of the H function

$$H(t)=H_0+H_1\exp\left(-\frac{t}{t_R}\right)\qquad(4)$$

The relaxation time and dynamical time scales as

$$t_R\propto\frac{1}{\Omega},\quad t_D\propto\frac{1}{N\Omega}\qquad(5)$$

Time evolution of H function is shown in Fig. 2 with various N and constant Ω and in Fig. 3 with

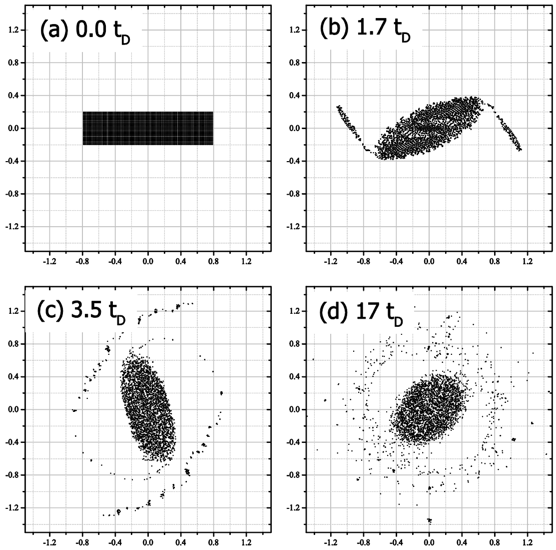

Fig. 1 Time evolution of the vortices is shown.

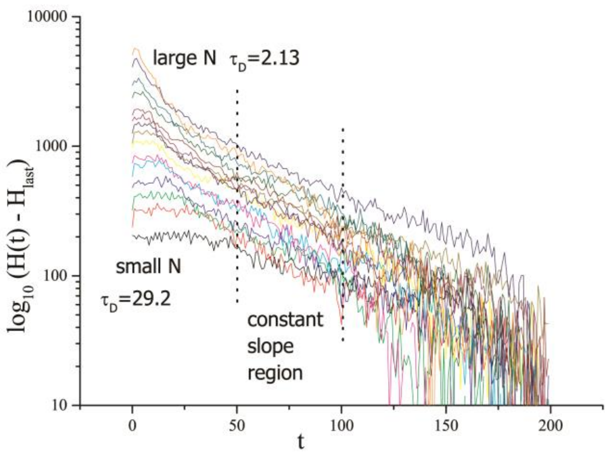

Fig. 2 Temporal evolution of the H function with variable N and constant Ω is shown.

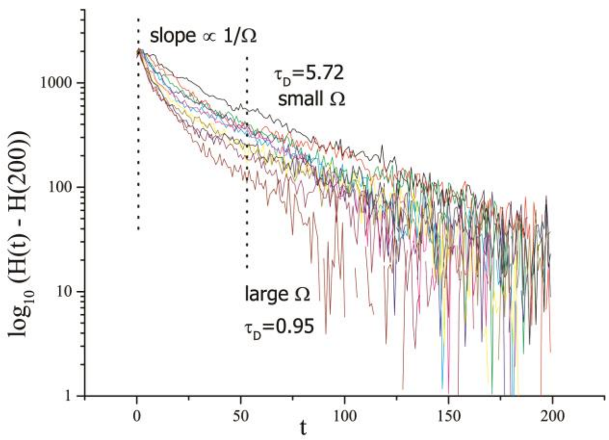

Fig. 3 Temporal evolution of the H function with variable Ω and constant *N* is shown.

various Ω and constant *N*.

As is shown in Eq. (5), relaxation time is independent of *N*. Thus the slopes in Fig. 2 should be constant. Between *T*=50 and 100, we can see the constant slopes. In the large *N* case, the initial steep slope gradually becomes gentle. This corresponds to the violent relaxation due to its short dynamical time. On the other hand, as the relaxation time is proportional to 1/Ω, the slopes in Fig. 3 should be changed according to the values of Ω. In this case, between *T*=0 and 40, the slopes change according to the value of Ω. The slopes against 1/Ω are plotted in Fig.4. As is expected by the theory, the slopes are proportional to 1/Ω.

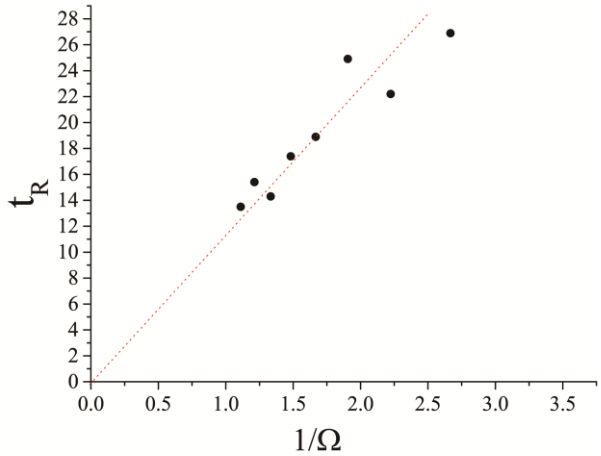

Fig. 4 Slope is plotted as a function of 1/Ω.

However, there are some cases that the steep slopes gradually lie down. It is possible that the slow relaxation driven by the collision term has two-stage relaxation. Namely, the relaxation speed slows down after the system profile approaches an axisymmetric one. This is due to the obtained collision term only represents $1/N$ factor. To understand the latter slower relaxation, we must evaluate the collision term including $1/N^2$ or higher effect.

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References

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