20pC1-3

Electromagnetic Waves Propagating in the External Plane Wave by High-Intensity Laser
高強度レーザーによる平面電磁波中を進行する電磁波

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We consider a new method of the perturbation in the framework of Schwinger's proper time method[1]. In the present study, we confirm the new method by calculating the vacuum polarization in crossed fields[2]. The vacuum polarization is a phenomenon that vacuum has a refractive index different from unity. The refractive index is derived for particular mode in the laser intensity of 10^{25} W/cm^2 at low-frequency limit. The goal of our study is to apply our new method to the varying electromagnetic field.

1. Introduction

By the recent progress of laser facilities, we will obtain the higher intensity of the laser facilities near future. The present maximum intensity of laser is order of 10^{22} W/cm^2. The next-generation laser facilities planned to get the intensity up to 10^{25} W/cm^2. One of such facilities in Japan is Gekko EXA[3]. The progress in the intensity of laser facilities can be profitable for study of the vacuum. The property of the vacuum changes with existence of the strong electromagnetic field in theory of quantum electrodynamics.

In quantum electrodynamics, it is thought that the quantum processes, which never occur in ordinary vacuum, occur in the strong electromagnetic field. For example, the vacuum creates the pair of an electron and a positron (pair creation) and have the anisotropy in the refractive index (vacuum polarization). There are thresholds of quantum processes for electromagnetic field called the critical field. For electric field, the critical field is 1.6×10^{19} V/cm and for magnetic field, the critical field is 4.4×10^{13} G. This threshold isn't strict and vacuum polarization is thought to occur below the threshold and pair of an electron and a positron can be created below the threshold by superimposing multiple electro- magnetic field[4].

The strong electromagnetic field in laser experiment fascinates not only fundamental physics but also astrophysics. There are some objects possessing strong magnetic field called magnetars. Magnetars are thought to possess extremely strong magnetic field of $\sim10^{14-15}$ G[5] and their giant flares are observed[6]. Although the energy source of the giant flare is thought to be their magnetic field, the mechanism of giant flare is not understood at this time. In such a strong magnetic field, the quantum processes have important role for their giant flare. The experiments of the strong magnetic field can't be realized because such a strong magnetic field can't be sustained. But when we think of electromagnetic field other than only magnetic field, we can experiment in laser experiments. Thus, the laser experiments is important for the fundamental study of magnetars.

The electromagnetic field of the laser is plane electromagnetic wave in nature and has variation of the field. Until now, the quantum processes in constant and homogeneous electromagnetic field is more often studied, however those in varying field isn't done so much. Particularly, the processes in plane wave field has never been done, so we should treat the plane wave when we consider laser experiments.

In our present study, we aim deriving of the vacuum polarization in laser field reflecting the frequency of the plane wave. For the preparation, we consider the vacuum polarization in crossed field(Fig. 1). The crossed field has the orthogonal electric and magnetic field($\mathbf{E}\perp\mathbf{B}$) and each field has same modulus($|\mathbf{E}|=|\mathbf{B}|$, where we use natural unit $c=\hbar=1$), and crossed field corresponds to low frequency limit of plane wave. The vacuum polarization in crossed field is derived in [1] by calculating Feynman diagram directly, which is often used in quantum electrodynamics. We use another method for the aim of treating the variation of the field.

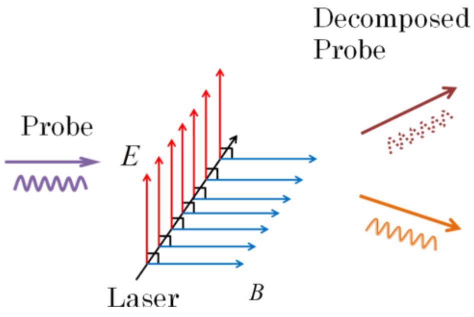

Fig.1. The schematic picture of birefringence in the crossed field

2. Method

The main purpose of the problem is to derive the quantum correction to Maxwell equation. We can write Maxwell equation as follows

$$\Box a_\mu = < j_\mu > \quad (1)$$

and the term $< j_\mu >$ is what to derive. The role of this term is current. To solve this problem, we use Schwinger's proper time method. In this method, the problem of quantum electrodynamics reduces to quantum mechanics by introducing the new parameter called proper time.

In [2], the only plane wave field is solved. In this problem, there are two fields, the strong external field and the probe field. We have to treat the probe field other than the external field. We assume the probe as the perturbation in the external field and solve it. The perturbation in the constant and homogeneous magnetic field is discussed in [7,8]. Although they solve the proper time evolution of the perturbation field, we solve the proper time evolution by the interaction picture in quantum mechanics.

3. Conclusion

Here, we think of the crossed field of field strength of f and the probe as the wave represented by

$$a_\mu = (0,0,0,a_3 \exp\{-i(\omega t - k_y y)\}), \quad (2)$$

that is the mode having the electric field along z-axis and the magnetic field along x-axis and propagating y-axis. We also assume that the crossed field strength as that of next-generation laser facilities and the probe as x-ray photon. In this case, the phase velocity of the probe is

$$v_p = 1 - \frac{14}{45}\frac{\alpha^2}{m^4}E^2 \quad (3)$$

and the refractive index of the vacuum is

$$n = 1 + \frac{14}{45}\frac{\alpha^2}{m^4}E^2 . \quad (4)$$

Where, $E = |\mathbf{E}| = |\mathbf{B}|$ and $\alpha = e^2/4\pi$ is fine structure constant.

We know that refractive index doesn't depend on probe photon energy, and this fact is valid if the condition $\omega E \ll 1$ is satisfied. The deviation of refractive index from unity is

$$n - 1 \approx 8.6 \times 10^{-8} \quad (5)$$

in case of 10^{25} W/cm^2 field.

4. Discussion

We derive the vacuum polarization in the crossed field by solving perturbation in interaction picture of proper time method. The conclusion is consistent to conclusion of [1]. In the future work, we expand the frequency of the external field. The first step is to treat the frequency is to solve the proper time problem of operators. Then we will get the solution of the plane wave field.

The crossed field is valid for external field of optical laser and probe of x-ray because the wavelength of the external field is much longer than probe wave. In case of the wavelength of the laser is x-ray, the external field for probe is varying and the crossed field limit is not valid and we should treat the variation of the plane wave field.

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