Sustainment mechanism of the solar tachocline  
太陽タコクラインの維持機構

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One of our goals is to understand the sustainment mechanism of the solar tachocline, a thin layer at the interface between the convection zone and the radiative zone. A mystery of the solar tachocline is that it is surviving against the momentum transport from the differentially rotating convection zone to the rigidly rotating radiative zone. In a simple hydrodynamics model, the momentum transport seems to be unavoidable due to the radiative spreading effect. Here in this talk, we explain the physical mechanism of the radiative spreading.

1. Introduction
One of the mysteries in solar physics is the tachocline, which is a sharp interface between the rigidly rotating radiative zone and the differentially rotating convection zone.

One of the riddles is why it is so thin. The underlying radiative zone is in a rigid rotation. (Incidentally, this is another mystery, when we remind ourselves the long viscous time compared to the age of the sun.) The momentum transport from the overlying convection zone to the radiative zone seems to be perfectly blocked by the tachocline. The momentum is expected to be conveyed due to the so-called radiative spreading process [1]. It is the thermal conductivity, rather than the viscosity, that plays the central role to convey the horizontal momentum along the vertical direction in this radiative spreading. [The Prandtl number in the tachocline is O(1E-6).]

While our final goal is to understand why the tachocline can survive against the radiative spreading, firstly in this talk, we try to elucidate the radiative spreading itself.

2. Setup
We consider a box region of stably stratified layer, a model of the radiative zone, with the cartesian coordinate system (x, y, z). The top boundary corresponds to the bottom of the convection zone. We assume the two-dimensionality of the system (∂_y = 0). The z-axis is the vertical (upward) direction and it is parallel to the rotation vector Ω. Ignoring high frequency modes such as the sound wave and the gravitational wave, time development appears only in the y-component of the equation of motion. In other words, we adopt the anelastic approximation, and the geostrophic (or heliostrophic) balance approximation. Taking the linear approximation, we obtain the following set of equations:

\[ \frac{\partial f_y}{\partial t} = -2\Omega f_x, \]  
\[ \frac{\partial f_y}{\partial z} = -2\Omega f_x, \]  
\[ \frac{\partial f_y}{\partial z} = -2\Omega f_x, \]  
\[ f_x = -\frac{\kappa g}{c_s T_0 N^2} \frac{\partial^2 T_1}{\partial z^2}. \]  

where \( p, \rho, T, f \) are pressure, mass density, temperature and mass flux, respectively. \( \kappa \) and \( N^2 \) are thermal conductivity and buoyancy frequency. Subscript 0 denotes the background field and 1 denotes the linear quantities.
3. Radiative spreading

Eq. (1) indicates the heliostrophic balance. When we apply a shear flow in the y-direction on the top boundary (see the upper part of Fig. 1), the pressure on the top boundary is redistributed to balance the Coriolis force; a cyclone or anti-cyclone is formed between each pair of the horizontal flow (see the lower part of Fig. 1).

The pressure modulation on the top boundary means the temperature modulation [see eq. (2) and Fig. 2(a)], and eventually causes the vertical flow due to the combination of eqs. (2) and (4). See Fig 2(b). The solenoidal condition of the mass flux in the anelastic condition leads to the x-component of the flow below the imposed y-flow. The Coriolis force [eq. (3)] accelerates the y-component there in the same directions as those imposed on the boundary. Thus the flow on the top boundary spreads down the stably stratified layer. This is the mechanism of the radiative spreading.

An interesting consequence of this process is that it naturally leads to the formation of the flow with helicity, or the helical flow lines as shown in Fig. 3. The implication of this helicity for the dynamo in MHD calculations would be an interesting point to be investigated in future.

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References