

Dynamics of “Dress of Charge” via Radiation Reaction

放射の反作用による電子衣のダイナミクス

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The laser intensity become higher year by year. In this regime, new physical process might be discovered. One of them is the dynamics of an electron’s charge to mass ration corrected from Fletcher-Millican’s. It is supposed by new equation of an electron’s motion with radiation reaction under ultra-high-intense laser field. In this conference, we will show the dynamics of quantum fluctuations as the dress of charge.

1. Introduction

Ultrahigh intense lasers are being planned and constructed [1]. What kind of physical processes can we observe in the regime of these laser intensities? We often discuss QED effects like pair creations/ annihilations by extreme intensities of 10^{24} -W/cm² [2]. This world will become new horizon in laser physics. Before reaching this physical process, we need to pass through the region of 10^{21-22} W/cm². In this regime, it is predicted that an electron will emit significant energy as light. Therefore, the motion of the electron needs to be corrected by the radiation feedback [3,4]. This is a basic physical process, named ‘radiation reaction’. Many readers may consider, that this is very obvious and there is no room for discussion. However, this radiation reaction remains as one of the difficult problems in physics. This standard theory was formulated by Lorentz [5], Abraham [6] and Dirac [7]. Therefore, the equation of motion with radiation reaction is named the Lorentz-Abraham-Dirac (LAD) equation [7].

$$m_0 \frac{d}{d\tau} w^\mu = -e(F_{\text{ex}}^{\mu\nu} + F_{\text{LAD}}^{\mu\nu})w_\nu \quad (1)$$

$$F_{\text{LAD}}^{\mu\nu} = -\frac{m_0 \tau_0}{ec^2} \left(\frac{d^2 w^\mu}{d\tau^2} w^\nu - \frac{d^2 w^\nu}{d\tau^2} w^\mu \right) \quad (2)$$

However, these equations have a mathematical difficulty which is named “run-away”. For instance, we consider the case without any external fields. In this case, $dw^\mu/d\tau \propto \exp(\tau/\tau_0) \rightarrow \infty$, it isn’t a solution for physics.

Recently, the new model of radiation reaction in quantum vacuum was proposed [8].

$$m_0 \frac{d}{d\tau} w^\mu = -e \frac{(F_{\text{ex}}^{\mu\nu} + F_{\text{LAD}}^{\mu\nu})}{(1 - \eta g_{\alpha\beta} g_{\beta\sigma} F_{\text{LAD}}^{\alpha\beta} F_{\text{LAD}}^{\rho\sigma})} w_\nu \quad (3)$$

Quantum vacuum is described by the Heisenberg–Euler Lagrangian density [9,10], it was an effective theory in low energy limit for photons propagation. The interesting point of this model is, radiation modulates the Fletcher-Millican’s charge to mass ratio, like,

$$\frac{Q}{M} = \frac{e}{m_0(1 - \eta g_{\alpha\beta} g_{\beta\sigma} F_{\text{LAD}}^{\alpha\beta} F_{\text{LAD}}^{\rho\sigma})} = \frac{e}{m_0} + \frac{\delta e}{m_0}, \quad (4)$$

$$\delta e = e \frac{\eta(1 - \eta g_{\alpha\beta} g_{\beta\sigma} F_{\text{LAD}}^{\alpha\beta} F_{\text{LAD}}^{\rho\sigma})}{1 - \eta(1 - \eta g_{\alpha\beta} g_{\beta\sigma} F_{\text{LAD}}^{\alpha\beta} F_{\text{LAD}}^{\rho\sigma})} \leq 0. \quad (5)$$

The Heisenberg-Euler model has been renormalized the 1-loop, δe is relate with dress of charge in QED. It is the method for the point on an electron.

Here, we consider the extension of this dress of charge. When we are able to define the LAD field in all of the Minkowski spacetime, we may see the behavior of the dress of electron’s charge. Since the basic physics is so-called, the photon-photon scattering which is one of the processes in nonlinear QED, this research will lead us to these study field beyond normal laser-plasma interactions.

2. Calculations

For calculation of electron’s charge, we are required to know an electron’s trajectory. Because an accelerated electron’s makes electric field in quantum vacuum and this electric field makes finally dress of electron’s charge. The electron’s trajectory is derived from Eq.(3). From the electron’s motion, we can calculate an electric field in the Minkowski spacetime. In this calculation, we need to use advanced and retarded field of an electric field. We can obtain these both fields by using Liénard–Wiechert potential. To use Liénard–

Wiechert potential, the electric “retarded” field is calculated [11].

$$\mathbf{E}(\mathbf{x},t) = -\frac{e}{4\pi\epsilon_0 c} \frac{\mathbf{n}(t') \times [\mathbf{n}(t') - \boldsymbol{\beta}(t')] \times \dot{\boldsymbol{\beta}}(t')}{[1 - \mathbf{n}(t') \cdot \boldsymbol{\beta}(t')]^3 |\mathbf{x} - \mathbf{x}_0(t')|} - \frac{e}{4\pi\epsilon_0} \frac{[1 - \boldsymbol{\beta}(t')^2][\mathbf{n}(t') - \boldsymbol{\beta}(t')]}{[1 - \mathbf{n}(t') \cdot \boldsymbol{\beta}(t')]^3 |\mathbf{x} - \mathbf{x}_0(t')|^2} \quad (6)$$

We show the simulation result of an electric field from an electron. The field is both retarded field F_{ret} (Fig.1) and advanced field F_{adv} (Fig.2). In this simulation, an electron interacts with the $10^{22}\text{W}/\text{cm}^2$ -class laser. From this result, we can obtain these field in Minkowski spacetime. Next, the definition of the LAD field is

$$F_{\text{LAD}}^{\mu\nu} = \frac{F_{\text{ret}}^{\mu\nu} - F_{\text{adv}}^{\mu\nu}}{2}, \quad (7)$$

therefore, we can define this field on all over the points in the Minkowski spacetime by following Eq.(7). After calculating F_{LAD} , the dress field (vacuum polarization)

$$\frac{M^{\mu\nu}}{\epsilon_0 c}(x) = -\eta(g_{\alpha\beta} g_{\rho\sigma} F_{\text{LAD}}^{\alpha\beta} F_{\text{LAD}}^{\rho\sigma}) F_{\text{LAD}}^{\mu\nu}(x) \quad (8)$$

$$M^{\mu\nu} = \begin{bmatrix} 0 & -P_x & -P_y & -P_z \\ P_x & 0 & M_z/c & -M_y/c \\ P_y & -M_z/c & 0 & M_x/c \\ P_z & M_y/c & -M_x/c & 0 \end{bmatrix} \quad (9)$$

becomes calculable, finally by calculating divergence of the dress field, we can obtain the charge density all over Minkowski spacetime as follows:

$$\rho_p(x) = -\partial_\mu M^{\mu 0} = -\nabla \cdot \mathbf{P} \quad (10)$$

This is the distribution of the dress of an electron’s charge.

We will show the simulation result of dress of electron’s charge in PLASMA CONFERENCE 2014.

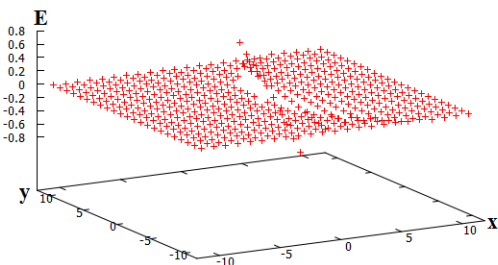


Fig 1. Retarded field of electric field's xy plane.

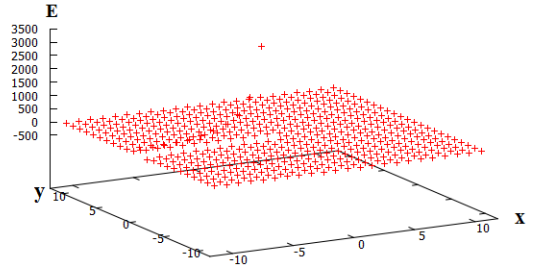


Fig.2. Advanced field of electric field's xy plane.

Acknowledgments

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References

- [1] G.A. Mourou, N.J. Fisch, V.M. Malkin, Z.Toroker, E.A. Khazanov, A.M. Sergeev, T.Tajima and B.Le Garrec, *Opt. Comm.* 720 (2012) **285**.
- [2] Report on the Grand Challenges Meeting, 27-28 April 2009, Paris.
- [3] J. Koga, *Phys. Rev. E* **70** (2004) 046502.
- [4] A. Zhidkov, J. Koga, A. Sasaki and M. Uesaka, *Phys. Rev. Lett.* **88** (2001) 18.
- [5] H. A. Lorentz, *The Theory of Electrons and Its Applications to the Phenomena of Light and Radiant Heat, A Course of Lectures Deliverd in Columbia Univ., New York, in March and April 1906*, 2nd edition (Teubner, Leipzig, 1916).
- [6] M. Abraham, *Theorie der Elektrizität: Elektromagnetische Theorie der Strahlung*, (Teubner, Leipzig, 1905).
- [7] P. A. M. Dirac, *Proc. Roy Soc. A* **167** (1938) 148.
- [8] K. Seto, S. Zhang, J. Koga, H. Nagatomo, M. Nakai and K. Mima, *Prog. Theor. Exp. Phys.* **2014**, 043A01 (2014).
- [9] W. Heisenberg and H. Euler, *Z. Phys.* **98**, 714 (1936).
- [10] J. Schwinger, *Phys. Rev.* **82**, 664 (1951).
- [11] K. H. Panofsky and M. Phillips, “Classical Electricity and Magnetism” (Addison-Wesley Pub. Co., 1961)