

## Maxwell equation of motion

マクスウェルの運動方程式

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The Bohr atomic model of a hydrogen atom is known well. The mechanism between an electron orbit and the light emitted from an atom was appreciably solved by the Bohr atomic model. The purpose of this paper is to show the "equation of motion" from the Maxwell equation. The improvement to the Bohr atomic model with the theory of relativity was tried. Moreover, the results of the study show a good approximation. The relativistic energy in the orbit of the electron in atom is calculated by energy function. These relations suggest to us a figure of the electron in atom. We think that shift to the angular moment from the image of the Planck constant and see the relation between the angular moment and the surrounding frequency energy.

### 1. Introduction

When guessing an atomic model, it is useful to take into consideration the hydrogen model of Bohr which is the fundamental atomic models. As for the hydrogen model of Bohr, the electron which has negative electric charge is moving with the surrounding on arbitrary orbits around the proton with positive electric charge.

However, in classic electricity and magnetism, it is generally expected. Since circular motion is accelerated motion, an electron emits light. An electron consumes the kinetic energy for the discharge, and it is mutually drawn according to the electrostatic force between a proton and an electron.

Furthermore, the electron which emitted light is drawn toward the proton with relatively large mass compared with the electron.

The purpose of this research is to show the orbit of the electron in the atom which used the equation of motion, and the relevance of an electromagnetic field.

By using the modified Maxwell equation of motion which added the time component, the equation of motion of the electron derives, and it performed the theory establishment of the optical discharge mechanism of low of Ritz in an atom.

Moreover, it clarified using the atomic model of Bohr about the relation between an optical discharge mechanism and an electron orbit.

This paper reports the example result about movement of the electron by the Maxwell equation of motion.

### 2. Maxwell equation

The conventional principle of conservation of energy  $E(r, v) = const.$  is equivalent to the time component of the Maxwell equation of motion in this paper, and an energy function is described by the following equations.

$$E(r, v) = \frac{m_e c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} e^{-\frac{keQ}{m_e c^2 r}} \quad (1)$$

$$= m_e c^2 + \frac{1}{2} m_e v^2 - \frac{keQ}{r} + \dots$$

Where  $m_e$  is an electron mass,  $c$  is the speed of light,  $v$  is the velocity of an electron,  $k$  is Coulomb constant,  $e$  is an electron charge,  $Q$  is a proton charge,  $r$  is the distance between a proton and an electron.

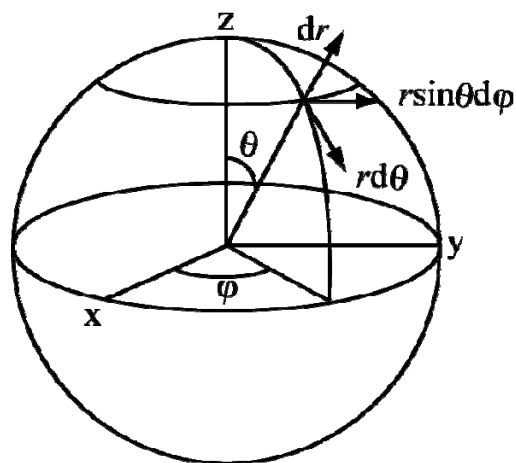


Fig.1. The spherical coordinate.

The electron revolve about a proton will be described by the following equations which is the space component of the Maxwell equation of motion.

$$m_e \frac{d^2 r}{d\tau^2} = -\frac{keQ}{c^2 r^2} \left( \frac{dct}{d\tau} \right)^2 + \frac{m_e}{r} \left\{ r \cosh \Omega \frac{d\varphi}{d\tau} - \left( r \frac{d\Omega}{d\tau} \right)^2 \right\} \quad (2)$$

Where  $\Omega$  is a parameter about rotation. In the case of a circular orbit  $d^2 r/d\tau^2 \equiv 0$  and  $d\Omega/d\tau \equiv 0$ .

The energy function in the circular orbit of the electron in an atom is described by the following formulas from the energy function of the formula (1) and the balance the low of Coulomb and the acceleration of the formula (2).

$$E(r) = \frac{m_e c^2}{\sqrt{1 - \left( \frac{v}{c} \right)^2}} e^{-\frac{keQ}{m_e c^2 r}} = m_e c^2 - \frac{keQ}{2r} + \dots \quad (3)$$

The calculation result of the formula (3) is shown.

$$r_0 = 2keQ/m_e c^2 \cong 5.6358807 \times 10^{-15} \text{ [m]}$$

In consideration of relativistic quantity of motion and a quantum condition, numerical analysis to a Bohr radius was conducted using Fortran. At the calculation results, we can get the following value.

$$r_1 \cong 5.29166 \times 10^{-11} \text{ [m]}, r \cong 3.9 \times 10^{-16} \text{ [m]}$$

It was a value like the half of the radius of a proton. Time to pass a radius at the speed of light is the following value.

$$t = \frac{r}{c} \cong 1.3 \times 10^{-24} \text{ [s]}, v = \frac{1}{t} \cong 7.6 \times 10^{23} \text{ [s}^{-1}\text{]}$$

Energy is following value.

$$E = h\nu \cong 3.1 \times 10^9 \text{ [eV]}$$

### 3. The angular moment and surrounding frequency energy

Electronic movement is mainly determined by the electric charge of a proton. The schematic view of movement of the electron in an atom is shown in Fig. 2. A horizontal axis is the x-axis, a vertical axis is the y-axis, an electron move a circular orbit at the speed  $v_y$  in the position of x, and the acceleration in this case is the time component of 4-dimensional force  $f_t = 0$ . And the space component will be described by the following formulas.

$$f_x = -P_x \frac{d\varphi}{dt}, P_x = \frac{m_e v_y}{\sqrt{1 - \left( \frac{v_y}{c} \right)^2}} \quad (4)$$

$$f_y = 0$$

$$f_z = -iP_z \frac{d\varphi}{dt}, P_z = \frac{m_e v_y}{\sqrt{1 - \left( \frac{v_y}{c} \right)^2}} \frac{v_y}{c}$$

Moreover, angular velocity is described by the following a formula.

$$\frac{d\varphi}{dt} = \frac{v_y}{r} = \sqrt{\frac{keQ}{m_e r^3}} \quad (5)$$

The surrounding energy is described by the following formulas.

$$E' = cP_z = h\nu_e, 2\pi r v_e = v_y, \lambda_e = 2\pi r \quad (6)$$

Where h is Planck constant,  $\nu_e$  is surrounding frequency,  $\lambda_e$  is wave length, r is Bohr radius.

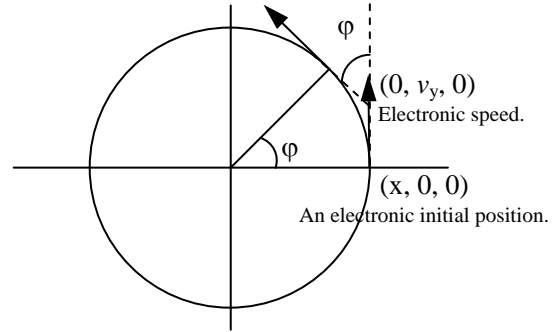


Fig.2. The movement of an electron.

### 4. Conclusion

By using the modified Maxwell equation, the relation between the optical discharge mechanism in an atom and the electron orbit were clarified using the atomistic model of Bohr.

It turned out that the energy and angular momentum of circumference pitch are related, and image of Planck constant may shift to angular momentum.

### References

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