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Evaluation of chaos-induced friction in Alfven system by using the projection operator method

射影演算子法を用いたアルヴェン系におけるカオス摩擦の評価

Yasuhiro Nariyuki[1],Makoto Sasaki[2],Nohiro Kasuya[2],Tohru Hada[3], Masatoshi Yagi[4]

成行泰裕[1]，佐々木真[2]， 糟谷直宏[2]， 羽田亨[3]， 矢木雅敏[4]

[1]*Faculty of Human Development, University of Toyama, 3190, Gofuku,Toyama City, Toyama 930-8555, Japan*

[1]富山大学人間発達科学部 〒930-8555　富山市五福3190

[2]*Research Institute for Applied Mechanics, Kyushu University, 6-1, Kasugakoen, Kasuga City, Fukuoka 816-8580, Japan*

[2]九州大学応用力学研究所 〒816-8580　春日市春日公園6-1

[3]*Faculty of Engineering Sciences, Kyushu University, 6-1, Kasugakoen, Kasuga City, Fukuoka 816-8580, Japan*

[3]九州大学大学院総合理工学研究院 〒816-8580　春日市春日公園6-1

[4]*Japan Atomic Energy Agency, 2-166, Omotedate, Obuchi, Rokkasho, Aomori 039-3212, Japan*

[4]日本原子力研究開発機構 〒039-3212　青森県上北郡六ヶ所村大字尾駮字表舘2-166

Alfven waves are ubiquitously observed in space and laboratory magnetized plasmas. The derivative nonlinear Schrodinger (DNLS) equation is known as a nonlinear evolution equation of Alfven waves. Hada *et al* (Phys. Fluid, 1990) derived a few degrees of freedom Alfven system from the DNLS by assuming the traveling wave solution and observed the chaotic behaviors of the system driven by the periodic external force. In the present study, the chaos-induced friction in the Alfven system is evaluated by using the Mori projection operator method.

Hada *et al*[1] derived ordinary differential equations from the driven derivative nonlinear Schrodinger (driven-DNLS) equation by assuming the stationary wave solution as follows:

$$\dot{b}_y - \nu\dot{b}_z = b_z(b_y^2 + b_z^2 - 1) - \lambda b_z + F\cos\theta, \qquad (1)$$

$$\dot{b}_z - \nu\dot{b}_y = -b_y(b_y^2 + b_z^2 - 1) + \lambda(b_y - 1) + F\sin\theta, \qquad (2)$$

where ν is the normalized dissipative scale length, λ is the phase speed of the wave, F is the amplitude of an external driver, and $\theta = \Omega t + \theta_0$, respectively. In this model, the Alfven wave is monochromatically amplified. Bifurcation diagrams of the system have been reported by the past studies[1][2].

We here assume $\langle b_y\rangle = \langle b_z\rangle = 0$, $\langle b_y b_z\rangle = 0$, and $\langle b_y b_y b_z\rangle = \langle b_y b_z b_z\rangle = 0$, where $\langle\ \rangle$ is the ensemble average. Let us rewrite eqs.(1)(2) as

$$\dot{b}_y - \nu\dot{b}_z = V_y - (\lambda + 1)b_z + F\cos\theta, \qquad (3)$$

$$\dot{b}_z - \nu\dot{b}_y = V_z + (\lambda + 1)b_y - \lambda + F\sin\theta, \qquad (4)$$

where $V_y = b_z(b_y^2 + b_z^2)$ and $V_z = -b_y(b_y^2 + b_z^2)$ are the nonlinear terms.

We consider the projection of the nonlinear terms ($\mathbf{V} = {}^t(V_y, V_z)$) by using the Mori projection operator method [3-6]. In what follows, the time dependence of the variables is specified such as $H(X(t))$. The projection of a variable $H(X(t))$ on the macrovariable $A \equiv A(t=0)$ is [3-6]

$$P(H(A(t))) = \langle H(A(t))A^*\rangle \langle AA^{*+}\rangle^{-1} A, \qquad (5)$$

where * indicates the Hermitian conjugate. In the same manner as past studies [5], we obtain

$$\mathbf{V}(\mathbf{b}(t)) = e^{\Lambda t}(P + Q)\mathbf{V}(\mathbf{b}), \qquad (6)$$

where $\mathbf{b} = {}^t(b_y, b_z)$ Λ is the evolution operator [5], and $Q = 1 - P$.

By using eqs.(5) and (6), we have [3-6]

$$V_y(\mathbf{b}(t)) = -\int_0^t \Gamma_{yy}(s) b_y(t-s)ds + L_y b_z(t) + r_y, \qquad (7)$$

$$V_z(\mathbf{b}(t)) = -\int_0^t \Gamma_{zz}(s) b_z(t-s)ds + L_z b_y(t) + r_z, \qquad (8)$$

where the second terms of eqs.(7)(8) ($L_y = (\langle b_z^4\rangle + \langle b_y^2 b_z^2\rangle)/\langle b_z^2\rangle$ and $L_z = -(\langle b_y^4\rangle + \langle b_y^2 b_z^2\rangle)/\langle b_y^2\rangle$) come from the first term of the right hand side in eq.(6)[3-6], while the first and third terms in the right hand side of eqs.(7)(8) come from the second term of the right hand side in eq.(6)[3-6]. The fluctuating force $\mathbf{r} = (r_y, r_z)$ is

$$\mathbf{r}(t) = e^{tQ\Lambda} QV(\mathbf{b}(t)), \qquad (9)$$

and the memory function is

$$\mathbf{\Gamma}(t) = \mathrm{diag}(\Gamma_{yy}, \Gamma_{zz}) = <\mathbf{r}(t)\mathbf{r}^* > < \mathbf{bb}^* >^{-1} . \qquad (10)$$

From eqs.(3)(4) and (7)(8), we obtain an evolution equation for time correlation function $\mathbf{C}$=<$\mathbf{b}$(t)$\mathbf{b}^*$> as [3-6]

$$\dot{C}_{yy} - v\dot{C}_{zy} = L_{t1}C_{zy} - \gamma_{yy}C_{yy} + C_{Fyy}, \qquad (11)$$

$$\dot{C}_{yz} - v\dot{C}_{zz} = L_{t1}C_{zz} + C_{Fyz}, \qquad (12)$$

$$\dot{C}_{zy} - v\dot{C}_{yy} = L_{t2}C_{yy} + C_{Fzy}, \qquad (13)$$

$$\dot{C}_{zz} - v\dot{C}_{yz} = L_{t2}C_{yz} - \gamma_{zz}C_{zz} + C_{Fzz}, \qquad (14)$$

where $\mathbf{C}$=<$\mathbf{F}$(t)$\mathbf{b}^*$>, $\mathbf{F}(t)={}^t(F\cos\theta,\ F\sin\theta)$, $L_{t1}=L_y-(\lambda+1)$, $L_{t2}=L_z+(\lambda+1)$, respectively. We here assume

$$\int_0^t \mathbf{\Gamma}(s)\mathbf{C}(t-s)ds \approx \boldsymbol{\gamma}\mathbf{C}(t) \qquad (15)$$

where $\boldsymbol{\gamma}$ =diag(γ_{yy}, γ_{zz}) is the chaos-induced friction coefficient [4]. We here also neglect the diagonal component of linear coefficients ($\mathbf{L_t}$). From eqs.(11)(12), we obtain

$$\ddot{C}_{yy} + a\dot{C}_{yy} + bC_{yy} = C_{Ft}, \qquad (16)$$

where

$$a = (\gamma_{yy} - v(L_{t1} + L_{t2}))/(1 - v^2), \qquad (17)$$

$$b = -L_{t1}L_{t2}/(1 - v^2), \qquad (18)$$

$$C_{Ft} = L_{t1}C_{Fzy} + v\dot{C}_{Fzy} + \dot{C}_{Fyy}. \qquad (19)$$

The direct numerical simulation of eqs.(1)(2) with the parameters λ=0.25, F=0.3, ν=0.19169, Ω=-1.0 is carried out by using the forth order Runge-Kutta method with Δt =10^{-3}. In this run, time evolution becomes chaotic [2](Fig.1). The ensemble average of correlation functions is numerically evaluated by using the time integral such as [3]

$$C_{yy}(t) = \frac{1}{N}\sum_{i=0}^{N-1} b_y(t + ih)b_y(ih). \qquad (19)$$

where h=0.1, N=37500 in the present run. For averaged values (t =0) such as <b_y^2>, we use $h=\Delta t$ and N=15000000. In this run, L_{t1}=0.108 and L_{t2}=-0.1063, respectively.

The numerical result shows that C_{Ft} can be written as $C_{Ft}=A_0\sin(t-\phi_0)$, where A_0=0.07855 and ϕ_0=2.752. Then, the special solution of eq.(19) becomes

$C_{yy}(t)=A_1\cos t'+A_2\sin t'$, (20)

where

$$A_1 = \frac{-aA_0}{(b-1)^2 + a^2}, \qquad A_2 = \frac{(b-1)A_0}{(b-1)^2 + a^2}, \qquad (21)$$

and $t'=t-\phi_0$. We here set $E=(A_1^2+A_2^2)^{0.5}$=0.0408, which corresponds to the energy of the frequency mode with ω=1.

By using the bisection method to find the solution of f(γ_{yy})=E-0.0408=0, we finally obtain the chaos-induced friction coefficient γ_{yy} =1.5920, A_1=-3.5019×10^{-2}, A_2=-2.0942×10^{-2}, respectively. Figure 2 shows the time dependence of C_{yy}. Although the phase of oscillation of eq.(20) (black line) agrees well with the one in the numerical result (gray line), the amplitude modulation due to the side band modes is not reproduced by the present approach. This is because of the simple approximation in eq.(15).

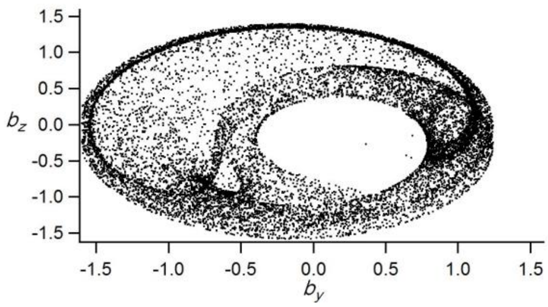

Fig.1. Hodogram (b_y-b_z).

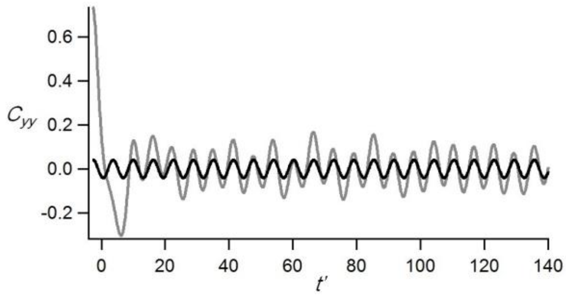

Fig.2. Time dependence of C_{yy}. Black and gray lines indicate eq.(20) and the numerical result, respectively. The chaos-induced friction coefficient γ_{yy} =1.5920.

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