Evaluation of chaos-induced friction in Alfven system by using the projection operator method

Yasuhiro Nariyuki¹, Makoto Sasaki², Nohiro Kasuya², Tohru Hada³, Masatoshi Yagi⁴

¹Faculty of Human Development, University of Toyama, 3190, Gofuku, Toyama City, Toyama 930-8555, Japan
²Research Institute for Applied Mechanics, Kyushu University, 6-1, Kasugakoen, Kasuga City, Fukuoka 816-8580, Japan
³Faculty of Engineering Sciences, Kyushu University, 6-1, Kasugakoen, Kasuga City, Fukuoka 816-8580, Japan
⁴Japan Atomic Energy Agency, 2-166, Omotedate, Obuchi, Rokkasho, Aomori 039-3212, Japan

Alfven waves are ubiquitously observed in space and laboratory magnetized plasmas. The derivative nonlinear Schrodinger (DNLS) equation is known as a nonlinear evolution equation of Alfven waves. Hada et al (Phys. Fluid, 1990) derived a few degrees of freedom Alfven system from the DNLS by assuming the traveling wave solution and observed the chaotic behaviors of the system driven by the periodic external force. In the present study, the chaos-induced friction in the Alfven system is evaluated by using the Mori projection operator method.

Hada et al[1] derived ordinary differential equations from the driven derivative nonlinear Schrodinger (driven-DNLS) equation by assuming the stationary wave solution as follows:

\[ \dot{b}_y - v b_y = b_z (b_z^2 + b_y^2 - 1) - \lambda b_z + F \cos \theta, \]  
\[ \dot{b}_z - v b_z = b_y (b_y^2 + b_z^2 - 1) + \lambda (b_y - 1) + F \sin \theta, \]  

where \( v \) is the normalized dissipative scale length, \( \lambda \) is the phase speed of the wave, \( F \) is the amplitude of an external driver, and \( \theta = \Omega t + \theta_0 \), respectively.

In this model, the Alfven wave is monochromatically amplified. Bifurcation diagrams of the system have been reported by the past studies[1][2].

We have assumed \( <b_y,> = <b_z,> = 0 \), \( <b_y b_z,> = 0 \), and \( <b_y b_y b_z,> = <b_y b_y b_z,> = 0 \), where \( < > \) is the ensemble average. Let us rewrite eqs.(1)(2) as

\[ \dot{b}_y - v b_y = V_y - (\lambda + 1) b_z + F \cos \theta, \]  
\[ \dot{b}_z - v b_z = V_z + (\lambda + 1) b_y - \lambda + F \sin \theta, \]  

where \( V_y := b_y (b_y^2 + b_z^2) \) and \( V_z := -b_z (b_y^2 + b_z^2) \) are the nonlinear terms.

We consider the projection of the nonlinear terms \( (V = (V_y, V_z)) \) by using the Mori projection operator method [3-6]. In what follows, the time dependence of the variables is specified such as \( H(X(t)) \). The projection of a variable \( H(X(t)) \) on the macrovariable \( A \equiv A(t=0) \) is [3-6]

\[ P(H(A(t))) = \langle H(A(t)) A^* \rangle \langle AA^* \rangle^{-1} A, \]  

where \( * \) indicates the Hermitian conjugate. In the same manner as past studies [5], we obtain

\[ V(b(t)) = e^{i t Q} (P + Q) V(b), \]  

where \( b = (b_y, b_z) \Lambda \) is the evolution operator [5], and \( Q = I - P \).

By using eqs.(5) and (6), we have [3-6]

\[ V_y(b(t)) = -\int_0^t \Gamma_y(s)b_y(t - s)ds + L_y(b_y(t) + r_y), \]  
\[ V_z(b(t)) = -\int_0^t \Gamma_z(s)b_z(t - s)ds + L_z(b_z(t) + r_z), \]  

where the second terms of eqs.(7)(8)

\( (L_y = \langle b_y^2 \rangle + \langle b_y^2 b_z^2 \rangle / b_z^2 \rangle \) and \( L_z = \langle b_y^2 \rangle + \langle b_y^2 b_z^2 \rangle / b_z^2 \rangle \) come from the first term of the right hand side in eq.(6)[3-6], while the first and third terms in the right hand side of eqs.(7)(8) come from the second term of the right hand side in eq.(6)[3-6]. The fluctuating force \( r = (r_y, r_z) \) is

\[ r(t) = e^{iQA} QV(b(t)), \]  

and the memory function is
\[ \Gamma(t) = \text{diag}(\Gamma_{yy}, \Gamma_{zz}) = <r(t)r^* > <bb^*>^{-1}. \quad (10) \]

From eqs.(3)(4) and (7)(8), we obtain an evolution equation for time correlation function \( C = <b(t)b^*> \) as [3-6]

\[ \dot{C}_{yy} - \nu C_{yy} = L_{1} C_{yy} - \gamma_{yy} C_{yy} + C_{yy}, \quad (11) \]
\[ \dot{C}_{zz} - \nu C_{zz} = L_{1} C_{zz} + C_{zz}, \quad (12) \]
\[ \dot{C}_{zy} - \nu C_{zy} = L_{2} C_{zy} + C_{zy}, \quad (13) \]
\[ \dot{C}_{zz} - \nu C_{zz} = L_{2} C_{zz} - \gamma_{zz} C_{zz} + C_{zz}, \quad (14) \]

where \( C = <F(t)b^*> \), \( F(t)=(F_{y} \cos \theta, F_{z} \sin \theta) \), \( L_{1} = L_{0} (\lambda + 1) \), \( L_{2} = L_{0} (\lambda + 1) \), respectively. We here assume

\[ \int_{0}^{\infty} \Gamma(s)C(t-s)ds \approx \gamma C(t) \quad (15) \]

where \( \gamma = \text{diag}(\gamma_{yy}, \gamma_{zz}) \) is the chaos-induced friction coefficient [4]. We here also neglect the diagonal component of linear coefficients \( (L_{0}) \).

From eqs.(11)(12), we obtain

\[ \dot{C}_{yy} + a \dot{C}_{yy} + b C_{yy} = C_{yy}, \quad (16) \]

where

\[ a = (\gamma_{yy} - \nu(L_{1} + L_{2}))/((1 - \nu^2), \quad (17) \]
\[ b = -L_{1}L_{2}/((1 - \nu^2), \quad (18) \]
\[ C_{yy} = L_{1}C_{yy} + \nu \dot{C}_{yy}, \quad (19) \]

The direct numerical simulation of eqs.(1)(2) with the parameters \( \lambda = 0.25, F = 0.3, \nu = 0.19169, \Omega = 1.0 \) is carried out by using the forth order Runge-Kutta method with \( \Delta t = 10^{-3} \). In this run, time evolution becomes chaotic [2](Fig.1). The ensemble average of correlation functions is numerically evaluated by using the time integral such as [3]

\[ C_{yy}(t) = \frac{1}{N} \sum_{i=0}^{N-1} b_{y}(t + ih)b_{y}(ih). \quad (19) \]

where \( h = 0.1, N = 37500 \) in the present run. For averaged values \( \langle t = 0 \rangle \) such as \( <b_{y}^*> \), we use \( h = \Delta t \) and \( N = 15000000 \). In this run, \( L_{1} = 0.108 \) and \( L_{2} = 0.1063 \), respectively.

The numerical result shows that \( C_{yy} \) can be written as \( C_{yy}(t) = A_{0}\sin(t + \phi) \), where \( A_{0} = 0.07855 \) and \( \phi = 2.752 \). Then, the special solution of eq.(19) becomes

\[ C_{yy}(t) = A_{1} \cos t + A_{2} \sin t, \quad (20) \]

where

\[ A_{1} = \frac{-aA_{0}}{(b-1)^{2} + a^{2}}, \quad A_{2} = \frac{(b-1)A_{0}}{(b-1)^{2} + a^{2}}. \quad (21) \]

and \( t' = t - \phi_{0} \). We here set \( E = (A_{1}^{2} + A_{2}^{2})^{0.5} = 0.0408 \), which corresponds to the energy of the frequency mode with \( \omega = 1 \).

By using the bisection method to find the solution of \( f(\gamma_{yy}) = 0.0408 = 0 \), we finally obtain the chaos-induced friction coefficient \( \gamma_{yy} = 1.5920, \quad A_{1} = 3.5019 \times 10^{-2}, \quad A_{2} = 2.0942 \times 10^{-2} \), respectively. Figure 2 shows the time dependence of \( C_{yy} \). Although the phase of oscillation of eq.(20) (black line) agrees well with the one in the numerical result (gray line), the amplitude modulation due to the side band modes is not reproduced by the present approach. This is because of the simple approximation in eq.(15).

References