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Experimental study on the dynamics of two vortex strings composed by a pure electron plasma

純電子プラズマで構成される２本の渦糸の運動に関する実験研究

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We have studied experimentally the dynamics of two vortices with different circulations using pure electron plasmas in a Malmberg trap. Two vortices with adequate circulation show stable orbits in 2D plane expected from the theory of 2D Euler fluid. However unstable orbits are observed when either of one of the vortices with a weak circulation loses its angular momentum. The discrepancy in dynamics between the magnetized electron plasma and Euler fluid is suppressed by adopting a stepwise trap potential on the boundary.

1.Introduction

In this study, we describe experiments on the dynamics of two string shaped pure electron plasma columns, trapped in a conducting cylinder under a strong homogeneous magnetic field. In this system, dynamics of the plasma within the two-dimensional (2D) guiding center approximation is equivalent to that of a 2D Eular fluid [1]. In this analogy, the vorticity is related to the electron density while the streamlines conform to the equipotential surfaces. Thus a string shaped electrons behaves as a vortex filament.

The motion of 2D vortex has been studied for over 100 years, and great achievements have accomplished in the field of simulation and theory. In experiments vortex dynamics using electron plasmas have been investigated on various topics which include a study on two discrete vortices with equal circulations [2,3]. However, experimental research on the vortices which have different circulation has not been conducted in terms of 2D Euler fluid probably because of a non-ideal effect on a trap potential in a typical Malmberg trap [4]. In this experiment, we precisely observe dynamics of two vortices with different circulation using electron plasmas and discuss the condition of the equivalence between 2D plasma system and 2D Eular fluid system.

2.Theory

In the system of 2D Euler fluid, two vortex strings inside of a cylindrical boundary have constant Hamiltonian written as,

$$H = \boldsymbol{\Gamma}_1^2 \log(1-r_1^2) + \boldsymbol{\Gamma}_2^2 \log(1-r_2^2) + \boldsymbol{\Gamma}_1 \boldsymbol{\Gamma}_2 \log\left(1 + (1-r_1^2)(1-r_2^2)/d^2\right), \quad (1)$$

$$d^2 = r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta_1 - \theta_2).$$

Here, r stands for a radial position of the vortex normalized by cylindrical boundary Rw, θ is an azimuthal angle, d is a distance between two vortices, $\boldsymbol{\Gamma}$ is circulation written as $eN/(\varepsilon_0 BL)$, N is a number of electrons, L is an axial length of plasma, and the subscripts 1 and 2 refer to each vortex. Angular momentum is also kept constant in the system written as

$$P_\theta = e B (N_1 r_1^2 + N_2 r_2^2)/2. \quad (2)$$

Due to the conservation of angular momentum, when a position of one vortex is determined, a position of another vortex is also determined. Together with the conservation of Hamiltonian the movement of two vortices can be predicted as an initial value problem in the system of Eular fluid.

3.Experimental Procedure

Malmberg trap shown in Fig.1 consists of electron sources, ring electrodes and a phosphor screen. Two electron strings are injected in the trap from two discrete electron sources. Negative voltages $\mathbf{V_c}$ applied to ring electrodes at both ends provide an axial confinement, and a homogeneous axial magnetic field provides a radial confinement. After any confinement time, by grounding the right-most ring electrode labeled with T11 (and T10 depending on a situation) and dumping the whole plasmas to the phosphor screen along magnetic field lines, we obtain a 2D image of two vortices and measure the number of electrons. Radial positions $r_{1,2}$, azimuthal angles $\theta_{1,2}$ and circulations $\boldsymbol{\Gamma}_{1,2}$ of two vortices are determined by this

Table I. Parameters of the experiment.
Confinement voltages are applied to the electrode labeled with T1, T2, T10 and T11.

	$\Gamma_{1,2}$ [m²/sec] ○ , ×	V_c [V] T1 and T11	V_c [V] T2 and T10
(a)	33 , 23	-80	0
(b)	9.7 , 3.4	-80	0
(c)	9.2 , 4.2	-80	-40

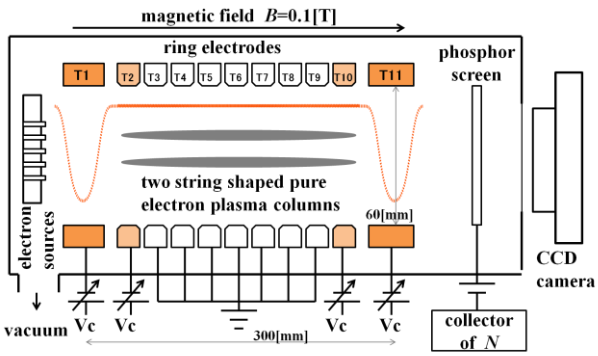

Fig 1. Schematic of a Malmberg trap.

measurements. By changing the confinement time, we can obtain a time evolution of two vortices' dynamics.

4. Result

We observe time evolution of 2D dynamics of two vortex strings for three different parameters shown in Table.I. Figure 2 represents orbits of two vortices in phase space, where we take radial positions and a difference of azimuthal angles as canonical valuables [3].

When both of vortices have large circulation of $\Gamma_{1,2}$ = 33, 23 , stable orbits of vortices are observed, corresponding to the Hamiltonian contours shown in Fig. 2(a). However unstable orbits are observed when vortices have weak circulation of $\Gamma_{1,2}$ = 9.7 , 3.4 (Fig. 2(b)). Interestingly, when a trap potential is modified into stepwise shape on the boundary by applying confinement voltage V_c of -80 V to ring electrodes labeled with T1, T11, and -40 V to T2, T10, the stable orbits of vortices corresponding to Hamiltonian contours are observed even if the circulation is small $\Gamma_{1,2}$ = 9.2 , 4.2 almost the same as the previous case of (b) shown in Fig. 2(c).

5. Discussion and Conclusion

There was a considerable decrease of angular momentum of 25 % from the initial state in the case of (b) whereas that is kept constant in other stable cases. As the result the orbit in the phase space deviates from the Hamiltonian contours.

In the case of (b), we observed a decrease of electron numbers for only a vortex with Γ_2 = 3.4 .This is because the excess effect on the vortex

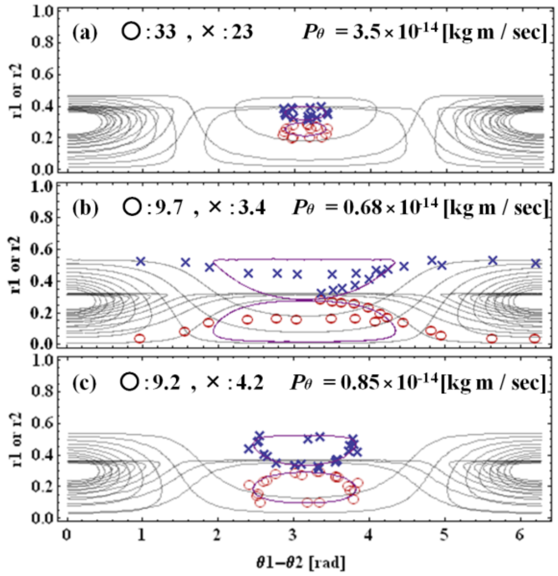

Fig 2. Orbits of two vortices in phase space for three different ratios of circulation shown in Table I. Solid lines represent Hamiltonian contours of each vortex assuming that angular momentum is kept constant. Hamiltonian and angular momentum are calculated from Eq.(1) and (2).

of an external electric field at both ends of the trap where the electrons reflect, which drives electrons to get additional $\boldsymbol{E}\times\boldsymbol{B}$ drift velocity [4]. As the result the angular momentum is not conserved and the dynamics deviates from 2D Euler theory in an isolated system.

This non-ideal effect of an external field can be suppressed by applying stepwise potential on the boundary shown in the case of (c) which indicates the stable dynamics expected from Euler fluid theory.

In summary, we have investigated 2D dynamics of two vortex filaments using pure electron plasma. Two vortices with adequate circulation show stable orbits in 2D plane expected from the theory of 2D Euler fluid. However unstable orbits are observed when the vortices have a weak circulation. The discrepancy in dynamics between the magnetized electron plasma and Euler fluid is suppressed by adopting a stepwise trap potential on the boundary.

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