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Gyrokinetic Field Theory of Classical, Neoclassical, and Turbulent Transport Processes in Toroidal Plasmas
トロイダルプラズマにおける古典・新古典および乱流輸送過程のジャイロ運動論的場の理論

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Effects of collisions on conservation laws for toroidal plasmas are investigated based on the gyrokinetic field theory. Associating the collisional system with a corresponding collisionless system at a given time such that the two systems have the same distribution functions and electromagnetic fields instantaneously, it is clarified how the collisionless conservation laws derived from Noether's theorem are modified by the collision term. Particle, energy, and toroidal momentum balance equations including collisional and turbulent transport fluxes are systematically derived. The ensemble-averaged transport equations of particles, energy, and toroidal momentum given in the present work are shown to include classical, neoclassical, and turbulent transport fluxes which agree with those derived from the conventional recursive formulation. It is found that the gyrokinetic collision operator and the momentum should retain correction terms of higher order in gyroradius expansion to reproduce the valid expression for the collisional transport of the toroidal momentum than for the collisional particle and energy transport.

1. Introduction

Recently, based on gyrokinetic theory and simulation, energy and momentum conservation in toroidal plasmas has been actively investigated to accurately describe transport processes determining energy and flow profiles. In gyrokinetic field theory, the gyrokinetic Vlasov equation, Poisson's equation, and Ampere's law are all obtained from the Lagrangian variational formulation, and conservation laws of energy and momentum for collisionless magnetized plasmas are derived by applying Noether's theorem [1-3]. However, since gyrokinetic equations for collisional systems are not derived from the variational principle, their conservation laws are not directly derived from Noether's theorem.

In this work, we investigate effects of collisions on conservation laws of particles, energy, and toroidal momentum for toroidal plasmas. The particle transport equation is obtained by the velocity-space integral of the gyrokinetic Boltzmann equation including Landau's collision operator represented in the gyrocenter coordinates. Energy, and toroidal momentum transport equations are derived by modifying Noether's method for collisional systems. Then, the ensemble-averaged transport equations of particles, energy, and toroidal momentum given in the present work contain all conventional expressions for classical, neoclassical, and turbulent transport fluxes.

2. Gyrokinetic Boltzmann Equation

The gyrokinetic Bolztmann equation for the gyrocenter distribution function F is written as

$$\left(\frac{\partial}{\partial t}+\frac{d\mathbf{Z}}{dt}\cdot\frac{\partial}{\partial\mathbf{Z}}\right)F(\mathbf{Z},t)=C^g \qquad (1)$$

where the gyrocenter coordinates $\mathbf{Z}$ = ($\mathbf{X}$, U, μ, ξ) consist of the gyrocenter position vector, parallel velocity, magnetic moment, and gyrophase, respectively, and C^g denotes the Landau's collision operator represented in terms of the gyrocenter coordinates. The gyrocenter coordinates $\mathbf{Z}$ are obtained by the transformation T of the particle coordinates $\mathbf{z}$ = ($\mathbf{x}$, $v_{||}$, μ_v, θ) as $\mathbf{Z} = T(\mathbf{z}) = \mathbf{z} + \Delta\,\mathbf{z}$. Then, in the gyrocener coordinates, the collision operator for collisions between particle species a and b is given by using pull-back operators T^* and T^{-1*} as

$$C^g_{ab}(F_a,F_b)=T_a^{-1*}C^p_{ab}(T_a^*F_a,T_b^*F_b)\,. \qquad (2)$$

Subscripts a and b representing particle species are omitted elsewhere unless they are necessary. In Eq.(2), C^p denotes the Landau collision operator represented in the particle coordinates and the distribution function f = T^*F in the particle coordinates is written as

$$f(\mathbf{z}) = F(\mathbf{Z}) = F(T(\mathbf{z})) = (T^*F)(\mathbf{z}) = F(\mathbf{z}+\Delta\mathbf{z})$$
$$= \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{i_1 \cdots i_n} \Delta z^{i_1} \cdots \Delta z^{i_n} \frac{\partial^n F(\mathbf{z})}{\partial z^{i_1} \cdots \partial z^{i_n}} \tag{3}$$
$$= F(\mathbf{z}) + \Delta\mathbf{z} \cdot \frac{\partial F(\mathbf{z})}{\partial \mathbf{z}} + \frac{1}{2} \Delta\mathbf{z}\Delta\mathbf{z} : \frac{\partial^2 F(\mathbf{z})}{\partial \mathbf{z} \partial \mathbf{z}} + \cdots .$$

3. Gyrocenter Continuity Equation

The gyrokinetic Boltzmann equation (1) can be rewritten as

$$\frac{\partial (D^g F)}{\partial t} + \frac{\partial}{\partial \mathbf{Z}} \cdot \left(D^g F \frac{d\mathbf{Z}}{dt} \right) = D^g C^g \tag{4}$$

where D^g denotes the Jacobian for the gyrocener coordinates. The right-hand side of Eq.(4) is written as a series expansion in $\Delta\mathbf{z}$,

$$D^g C^g_{ab}(F_a, F_b)$$
$$= \left[\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \sum_{i_1 \cdots i_n} \frac{\partial^n}{\partial z^{i_1} \cdots \partial z^{i_n}} \left[\Delta z^{i_1} \cdots \Delta z^{i_n} D^p C^p_{ab}(f_a, f_b) \right] \right]_{\mathbf{z}=\mathbf{Z}}$$
$$= \left[D^p C^p_{ab}(f_a, f_b) + \frac{\partial}{\partial \mathbf{z}} \cdot \left(\Delta\mathbf{z}\, D^p C^p_{ab} \right) + \frac{1}{2} \frac{\partial^2}{\partial \mathbf{z} \partial \mathbf{z}} : \left(\Delta\mathbf{z}\, D^p C^p_{ab} \right) + \cdots \right]_{\mathbf{z}=\mathbf{Z}} \tag{5}$$

where D^p represents the Jacobian for the particle coordinates. Integrating Eq.(4) over the (U, μ, ξ)-space immediately yields the gyrocenter continuity equation,

$$\frac{\partial n^{(gc)}}{\partial t} + \frac{\partial}{\partial \mathbf{X}} \cdot \left(\Gamma^{(gc)} + \Gamma^C \right) = 0 \tag{6}$$

where

$$n^{(gc)} = \int dU \int d\mu \int d\xi\, D^g F,$$
$$\Gamma^{(gc)} = \int dU \int d\mu \int d\xi\, D^g F \frac{d\mathbf{X}}{dt} \tag{6}$$

are the gyrocenter density and the gyrocenter flux, respectively, and

$$\Gamma^C = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)!} \sum_{i_1 \cdots i_n} \frac{\partial^n}{\partial X^{i_1} \cdots \partial X^{i_n}} \left[\int d^3v\, \Delta\mathbf{x} \Delta x^{i_1} \cdots \Delta x^{i_n} C^p \right]_{\mathbf{x}=\mathbf{X}} \tag{7}$$
$$= \left[\int d^3v\, \Delta\mathbf{x}\, C^p \right]_{\mathbf{x}=\mathbf{X}} + \cdots$$

represents the classical particle flux due to collisions and finite gyroradius $\boldsymbol{\rho} = -\Delta\mathbf{x}$. It is also found that the ensemble average of the gyrocenter flux $\Gamma^{(gc)}$ consists of the neoclassical and turbulent particle fluxes. Thus, the ensemble average of Eq.(6) coincides with the particle transport equation including the classical, neoclassical, and turbulent particle fluxes derived from the conventional recursive formulation.

4. Conservation of Energy and Toroidal Momentum

In Ref.[3], conservation laws of energy and toroidal momentum for collisionless axisymmetric toroidal plasmas are derived from Noether's theorem based on the gyrokinetic field theory. Associating the collisional system with a corresponding collisionless system at a given time such that the two systems have the same distribution functions and electromagnetic fields instantaneously, it is shown how the collisionless conservation laws derived from Noether's theorem are modified by the collision term. The resultant transport equations for energy and toroidal momentum take the flux-surface-averaged form,

$$\frac{\partial}{\partial t} \left(V' \left\langle \delta G_0 \right\rangle \right) + \frac{\partial}{\partial s} \left(V' \left\langle \left(\delta\mathbf{G} - \delta G_0 \mathbf{u}_s \right) \cdot \nabla s \right\rangle \right) = V' \left\langle \delta K_0 \right\rangle \tag{7}$$

where terms in the left-hand side are defined in Ref.[3] and the right-hand side is given from the collision term. We find that the classical fluxes of energy and toroidal momentum are naturally derived from the collision term and that the ensemble-averaged fluxes of energy and toroidal momentum are both divided into the neoclassical and turbulent parts. It is also noted that the gyrokinetic collision operator and the momentum should retain correction terms of higher order in gyroradius expansion to reproduce the valid expression for the collisional transport of the toroidal momentum than for the collisional particle and energy transport.

5. Summary

Effects of collisions on conservation laws for toroidal plasmas are investigated based on the gyrokinetic field theory. Particle, energy, and toroidal momentum balance equations including collisional and turbulent transport fluxes are systematically derived. The ensemble-averaged transport equations of particles, energy, and toroidal momentum given in the present work contain classical, neoclassical, and turbulent transport fluxes which agree with those derived from the conventional recursive formulation.

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