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Density-variance effects in reconnection shock
リコネクション衝撃波での密度揺動効果

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Density-variance effects on the turbulence transports are investigated in magnetohydrodynamic (MHD) turbulence with special emphasis on its relevance to the fast reconnection. The expression for the turbulent electromotive force (EMF), as well as the Reynolds and turbulent Maxwell stresses, turbulent mass and heat fluxes, is derived. Misalignment of the mean magnetic field from the gradient of mean density contributes to the EMF in the presence of the density variance, which is ubiquitous in the near shock region. It is shown that this density variance effect gives the enhancement of turbulence in the foreshock (upstream) region of the magnetic reconnection.

1. Introduction

Magnetic reconnection is ubiquitous in plasma phenomena. It is believed to be the cause of the energetic eruption phenomena such as flares on the surface of the Sun, and of the auroral substorms in the Earth's magnetosphere. Magnetohydrodynamic (MHD) slow shock is known to contribute to fast reconnection by changing the geometry of the reconnection magnetic fields [1]. Stochastic motion of the magnetic field has also been considered to be relevant to enhancing the magnetic reconnection rate [2-5]. From the viewpoint of turbulence theory and modeling, the shock–turbulence interaction is one of the most challenging problems. In addition to the turbulence influence on the shock strength, the spatiotemporal evolutions of turbulence intensity and vortical structures should be determined by the shock.

A large gradient of mean density induces a large density fluctuation, which is represented by the density variance $\langle \rho'^2 \rangle$ (ρ': density fluctuation, $\langle \cdots \rangle$: mean). In this work, we investigate the turbulence correlations in the inhomogeneous compressible MHD turbulence. We focus our argument on the roles of density variance in the turbulent electromotive force.

2. Mean-field equations

We divide field quantities into the mean and the deviation from it as

$$\rho = \overline{\rho} + \rho', \mathbf{u} = \mathbf{U} + \mathbf{u}', \mathbf{u} = \mathbf{B} + \mathbf{b}', q = Q + q' \quad (1)$$

where ρ is the density, $\mathbf{u}$ the velocity, $\mathbf{b}$ the magnetic field, and q the internal energy. The capitalized letters denote the mean while the primed ones the fluctuation ($\overline{\rho}$: mean density). In order to derive the density-variance effects more explicitly, we adopt the ensemble or Reynolds average rather than the mass-weighted or Favre average. Substitution of Eq. (1) into the fundamental MHD equations yields the mean-field equations. The mean magnetic induction equation is written as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \nabla \times \langle \mathbf{u}' \times \mathbf{b}' \rangle + \eta \nabla^2 \mathbf{B}. \quad (2)$$

In the mean-field equations, the fluctuation effects appear in the form of turbulence correlations such as the Reynolds stress $\langle \mathbf{u}'\mathbf{u}' \rangle$, the turbulent Maxwell stress $\langle \mathbf{b}'\mathbf{b}' \rangle$, the turbulent electromotive force $\langle \mathbf{u}' \times \mathbf{b}' \rangle$, the turbulent mass flux $\langle \rho' \mathbf{u}' \rangle$, and the turbulent energy flux $\langle q' \mathbf{u}' \rangle$.

3. Turbulent electromotive force

With the aid of a closure theory of inhomogeneous MHD turbulence, the expressions for the turbulence correlations are derived. For instance, the turbulent electromotive force (EMF) is expressed as

$$\langle \mathbf{u}' \times \mathbf{b}' \rangle = -\beta \nabla \times \mathbf{B} + \alpha \mathbf{B} + \gamma \nabla \times \mathbf{U}$$
$$- \chi_\rho \nabla \overline{\rho} \times \mathbf{B} - \chi_q \nabla Q \times \mathbf{B} - \chi_{\rm D} (D\mathbf{U}/Dt) \times \mathbf{B}. \quad (3)$$

Here, β, α, γ, χ_ρ, χ_q, and $\chi_{\rm D}$ are the transport coefficients, which are expressed in terms of the turbulent energy, helicity, cross helicity, and density variance as

$$\beta = \tau \langle \mathbf{u}'^2 \rangle + \tau \langle \mathbf{b}'^2 \rangle / \mu_0 \overline{\rho}, \quad (4a)$$

$$\alpha = \tau \langle -\mathbf{u}' \cdot \boldsymbol{\omega}' \rangle + \tau \langle \mathbf{b}' \cdot \mathbf{j}' \rangle / \mu_0 \overline{\rho}, \quad (4b)$$

$$\gamma = \tau \langle \mathbf{u}' \cdot \mathbf{b}' \rangle, \quad (4c)$$

$$\chi_\rho = (\gamma_{\rm s} - 1)^2 \mu_0^{-1} \tau_{\rm s} (Q/\overline{\rho}) (\langle \rho'^2 \rangle / \overline{\rho}^2), \quad (4d)$$

$$\chi_Q = (\gamma_{\rm s} - 1) \mu_0^{-1} \tau_{\rm s} (\langle \rho'^2 \rangle / \overline{\rho}^2), \quad (4e)$$

$$\chi_D = \mu_0^{-1} \tau_{\rm s} (\langle \rho'^2 \rangle / \overline{\rho}^2), \quad (4f)$$

where τ and $\tau_{\rm s}$ are appropriate timescales. Here, the β-, α- and γ-related terms represent the turbulent magnetic diffusivity, helicity and cross-helicity effects, respectively, which are present even in the incompressible case [6,7]. On the other hand, the fourth to sixth terms have no counterparts in the incompressible case. Note that all of χ_ρ, χ_ρ, and χ_ρ depend on the density variance, suggesting that these terms are relevant only in the case of strong compressible turbulence.

We focus on the effect of large-scale density condensation/stratification and confine our argument into the fourth or χ_ρ-related term in Eq. (3). The density variance equation is given by

$$\frac{D}{Dt}\left\langle \rho'^2 \right\rangle = -2\left\langle \rho' \mathbf{u}' \right\rangle \cdot \nabla\overline{\rho} - 2\left\langle \rho'^2 \right\rangle \nabla\cdot\mathbf{U} \qquad (5)$$

($D/Dt=\partial/\partial t+\mathbf{U}\cdot\nabla$). This shows that a large gradient of mean density leads to a large density variance. The equations for the fluctuation velocity and magnetic field are written as

$$\partial\mathbf{u}'/\partial t = -(\gamma_{\rm s}-1)(q'/\overline{\rho})\nabla\overline{\rho} + \cdots$$
$$= (\gamma_{\rm s}-1)^2\tau_q(Q/\overline{\rho})(\nabla\cdot\mathbf{u}')\nabla\overline{\rho} + \cdots\,, \qquad (6)$$
$$\partial\mathbf{b}'/\partial t = -(\nabla\cdot\mathbf{u}')\mathbf{B} + \cdots\,. \qquad (7)$$

In Eq. (6), use has been made of the simplest expression for the internal energy fluctuation:

$$q' = -(\gamma_{\rm s}-1)\tau_q Q\nabla\cdot\mathbf{u}'. \qquad (8)$$

It follows from Eqs. (6) and (7) that the turbulent electromotive force is expressed as

$$\langle\mathbf{u}'\times\mathbf{b}'\rangle/\tau \simeq (\gamma_{\rm s}-1)(1/\overline{\rho})\langle q'\nabla\cdot\mathbf{u}'\rangle\nabla\overline{\rho}\times\mathbf{B}$$
$$= (\gamma_{\rm s}-1)^2(\tau_q/\tau_\rho)(\langle\rho'^2\rangle/\overline{\rho}^2)(Q/\overline{\rho})\mathbf{B}\times\nabla\overline{\rho}, \quad (9)$$

where use has been made of Eq. (8) and the simplest expression for the density fluctuation:

$$\rho' = -\tau_\rho\overline{\rho}\nabla\cdot\mathbf{u}'. \qquad (10)$$

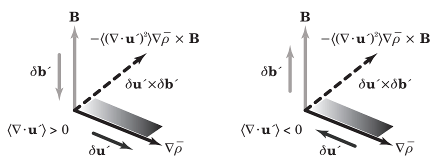

Fig. 1: Turbulent electromotive force due to the misalignment of the mean magnetic field from the gradient of mean density. Cases for dilatation (left) and contraction (right).

Equation (9) is equivalent to the fourth term in Eq. (3) with Eq. (4d). This expression shows that, irrespective of the sign of $\langle\nabla\cdot\mathbf{u}'\rangle$ (dilatation or contraction), the density variance will contribute to the turbulent EMF in the direction of $\mathbf{B}\times\nabla\overline{\rho}$. See Fig. 1.

4. Density variance effects in reconnection

We consider the roles of the density variance in the magnetic reconnection from the viewpoint of turbulent transport. Slow MHD shock waves are expected to contribute to the fast reconnection. Since shock waves are always associated with a large density variation, the density variance must be ubiquitous in the magnetic reconnection. The evolution equation of the turbulent kinetic energy $\langle\mathbf{u}'^2\rangle/2$ is given as

$$\frac{D}{Dt}\frac{1}{2}\left\langle\mathbf{u}'^2\right\rangle = -\frac{1}{2\overline{\rho}}\left\langle\mathbf{u}'\times\mathbf{b}'\right\rangle\cdot\mathbf{J} - \left\langle u'^a u'^b\right\rangle\frac{\partial U^a}{\partial x^b}$$
$$+\frac{1}{2\mu_0\overline{\rho}}\left\langle u'^a b'^b\right\rangle\left(\frac{\partial B^b}{\partial x^a}+\frac{\partial B^a}{\partial x^b}\right) + R_{\rm u}, \qquad (11)$$

where $R_{\rm u}$ denotes the terms arising from the turbulent mass flux $\langle\rho'\mathbf{u}'\rangle$, the turbulent heat fluxes $\langle q'\mathbf{u}'\rangle$, the dissipation and transport rates, whose expressions are suppressed here. In the upstream region, where the gradient of the mean density is positive in the direction to downstream and the magnetic field is in the direction vertical to the inflow direction (Fig. 2). Then we have a contribution to the EMF parallel to the original reconnection current. Substitution of Eq. (9) into Eq. (11) gives an enhancement of the turbulent energy production in the upstream region. On the other hand, in the downstream region, where the magnetic field and the gradient of mean density are aligned each other, there is no contribution from this effect.

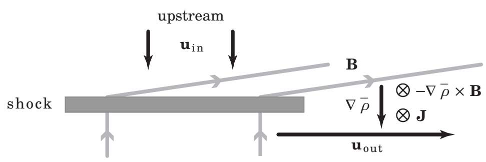

Fig. 2: Density variance effect in a reconnection slow shock.

In the slow shocks in magnetic reconnection, the turbulent EMF due to the mean density variation enhances the turbulence generation in the foreshock (upstream) region but no effects in the aftershock (downstream) region.

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