

The Dynamo Effect in Laboratory and Astrophysical Plasmas

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The dynamo effect manifests itself very differently in laboratory and astrophysical plasmas, but is described in this talk from a common perspective. We focus here on two examples: the reversed-field pinch (RFP), which is magnetically dominated, and the accretion disk, which is flow-dominated. In the RFP, the dominant instability is the tearing mode, whereas the accretion disk is assumed to be unstable with respect to the magnetorotational instability (MRI). The role of magnetic helicity transport is discussed, and the dynamo effect is calculated from variants of quasi-linear theory.

1. Introduction

Large-scale magnetic fields have been observed in widely different types of astrophysical objects, such as planets and stars, as well as accretion disks and galaxies. The source of such fields is largely attributed to the dynamo effect, which has stimulated an extensive search for models in which large-scale magnetic are self-generated from turbulence and sustained despite the presence of dissipation. The point of departure of most theoretical and computational studies of this problem is magnetohydrodynamics (MHD), represented by intrinsically nonlinear equations that describe the self-consistent evolution of a magnetized fluid. A standard approach to the problem is mean-field theory, in which a fluctuation-induced electromotive force parallel to the mean magnetic field is obtained from the vector product of flow and magnetic field fluctuations. This is often referred to as the so called α -effect. While kinematic dynamo theory [1] predicts the existence of the α -effect in astrophysical settings given a complex (often turbulent) velocity field, its magnitude and saturation in a fully nonlinear self-consistent theory remains a subject of significant controversy.

In this talk, we will discuss the dynamo effect as it manifests itself in two very different physical contexts. One example is a magnetically dominated self-organized laboratory plasma, the reversed-field pinch (RFP), in which the dynamo effect is instrumental in converting one type of flux to another by the intervention of tearing instabilities. The second example is a flow-driven accretion disk, which too exhibits self-organization, and where there is compelling evidence from several MHD simulations that a large-scale magnetic field is produced and sustained by the nonlinear evolution of the

magnetorotational instability (MRI). In both cases, we will discuss the successful applications of variants of quasilinear theory and the role of magnetic helicity transport.

2. The Reversed-Field Pinch Dynamo

There are numerous examples of highly successful RFP experiments in Europe, Japan, and the United States (e.g., see [2] and other references therein) in which there is strong experimental evidence that a turbulent plasma relaxes on the fast time scales of tearing instabilities to a state of minimum energy subject to the (approximate) conservation of total magnetic helicity, broadly consistent with Taylor's relaxation theory [3]. The eventual sustainment of these relaxed states on the longer time scales of resistive diffusion requires a dynamo effect mediated by tearing modes, which has been the subject of several theoretical investigations [4, 5]. It has been shown in these studies, based on essentially quasi-linear analyses of tearing modes, that the turbulent dynamo field parallel to the mean magnetic field can be written in the form,

$$\epsilon_{\parallel} = \frac{\mathbf{B}}{B^2} \cdot \nabla \cdot \left(\kappa^2 \nabla \frac{\mathbf{J} \cdot \mathbf{B}}{B^2} \right), \quad (1)$$

where \mathbf{B} is the magnetic field, \mathbf{J} is the current density, and κ^2 is a positive definite functional. This form of the dynamo field, now known as "hyperresistivity", preserves the total magnetic helicity and dissipates magnetic energy. It does not amplify mean magnetic fields in the manner attributed to the astrophysical dynamo, but in the presence of a source of external helicity injection, supports a mean-field RFP in a state close to but different from the Taylor state. In this talk, we will present the first direct numerical simulations (DNS) of hyperresistivity [6], which tests directly the predictions of the analytical theories [4-6].

3. The Accretion Disk Dynamo

Since the seminal work of Balbus and Hawley [7], the MRI has emerged as promising explanation for the observed momentum transport in accretion disks. In particular, the nonlinear development of the instability has been shown to lead to sustained turbulence and dynamo action in both local shearing box (e.g., [8]) and global nonlinear simulations (e.g., [9]). Despite substantial concerns about convergence of MRI turbulence simulations with dissipation parameters, it seems such results are relatively robust, but a thorough understanding of the dynamo mechanism is lacking. In this talk, we will report on some recent theoretical developments [10] that appear to be promising in elucidating the character of the underlying instabilities and the nonlinear MRI dynamo. The first insight stems from the recognition that conventional linear eigemode analyses fail to capture an essential aspect of the MRI. Since the underlying operator describing the MRI is not self-adjoint, what is needed is a rigorous non-modal analysis, which demonstrates that the shearing MRI wave energy can grow at the maximum rate $-d\Omega/d\ln r$, for *any* choice of azimuthal and vertical wavenumbers. (Here Ω is the angular speed and r is the radial coordinate.) By demonstrating that fast linear growth is possible at all wavenumbers, these results suggest that nonmodal linear physics could play a fundamental role in MRI turbulence.

The second important insight emerges from studying MRI turbulence and dynamo using novel quasi-linear statistical simulation methods [11,12], which have proven to be successful in a variety of geophysical and astrophysical applications. Motivated by the idea that strongly nonmodal growth is possible at all scales due to nonmodal effects, our quasi-linear approximation involves neglecting almost all of the fully nonlinear interactions in the system (leading to a turbulent cascade). Remarkably, despite the strongly reduced nonlinearity, we demonstrate that the qualitative dependence on dissipation parameters (specifically, the Prandtl number) is the same as fully nonlinear MRI turbulence. Furthermore, we see two important bifurcations---the first marking the onset of a dynamo instability of the background turbulence, and the second transition to a strongly time-dependent state. These two bifurcations bear a strong qualitative resemblance to the transitions seen in hydrodynamic plane Couette flow in which the second transition is associated with self-sustaining behavior [11].

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