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Comparison of Push-Forward Representations of Particle Flux in the Standard Electrostatic Gyrokinetic Model

標準ジャイロ運動論モデルにおける粒子フラックスの押し出し表現の比較

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Particle flux originally defined by an integral in terms of a particle distribution function and particle coordinates can be represented as an integral in terms of a gyro-center distribution function and gyro-center coordinates, which is called a push-forward representation of particle flux. Two formally different representations are possible in the standard gyrokinetic model. To obtain the polarization flux in the particle flux representation, a higher order solution for the scalar function generating the gyro-center transformation is needed for the conventional representation in which the gyro-center part appears as the pull-back transformation of the gyro-center distribution function. On the other hand, the lowest order solution is sufficient for the other one in which the pull-back transformation of the distribution function does not appear.

1. Introduction

The standard gyrokinetic model is formulated by a two-step phase space transformation from particle phase space to gyro-center phase space which consists of the guiding-center transformation and the following gyro-center transformation [1]. Any particle fluid moment can be represented as an integral in terms of a gyro-center distribution function and gyro-center coordinates, which is called a push-forward representation. There exist two formally different representations in the standard model [2]. The two representations differ in the gyro-center transformation part which contains effects of electrostatic potential fluctuations. In the representation conventionally used to derive the gyrokinetic Poisson equation, the gyro-center transformation part appears as the *pull-back* transformation of the gyro-center distribution function and it is separated from the guiding-center part. In contrast, they are combined in the other representation in which the pull-back transformation of the gyro-center distribution function does not appear. Hence, this is the more straightforward or pure push-forward representation. The pure representation is used for showing the correspondence between a gyrokinetic model for flowing plasmas and the standard model [3,4] and for constructing a high-order gyrokinetic model [5].

In the standard electrostatic model, the gyro-center transformation and its associated transformations are described by the Poisson brackets with scalar generating functions. Only the lowest order term of the generating function at first order S_1 is considered to derive the polarization density term in the gyrokinetic Poisson equation. While the polarization density appears in the gyrokinetic Poisson equation, the polarization drift does not appear as the gyro-center drift in the standard gyrokinetic model. As a consequence, the polarization drift flux is not included in the gyro-center flux. The polarization drift flux is recovered through the push-forward representation of particle flux. A disparity between the two representations becomes manifest in deriving the polarization flux in the push-forward representation of particle flux.

2. Conventional representation

Particle flux at position $\mathbf{r}$ is defined by

$$\boldsymbol{\Gamma}(\mathbf{r}) \equiv \int d^3\mathbf{x} d^3\mathbf{v} f(\mathbf{x},\mathbf{v})\delta^3(\mathbf{x}-\mathbf{r}) \qquad (1)$$

where f is the particle distribution function. The conventional push-forward representation of the particle flux is given by [6]

$$\boldsymbol{\Gamma}(\mathbf{r}) = \int d^6\mathbf{Z}\,\mathcal{J}\left[\mathrm{T}_{\mathrm{GC}}^{-1}\mathbf{v}\right](\mathbf{Z})\mathrm{T}_{\mathrm{Gy}}^{*}F\delta^3\left(\left[\mathrm{T}_{\mathrm{GC}}^{-1}\mathbf{x}\right](\mathbf{Z})-\mathbf{r}\right) \qquad (2)$$

where $\mathbf{Z}=(\mathbf{X},U,\mu,\xi)$ is the gyro-center coordinates, $\mathrm{T}_{\mathrm{Gy}}^{*}F$ is the pull-back transformation of the gyro-center distribution function associated with the gyro-center transformation, $\mathcal{J}$ is the Jacobian of the guiding-center transformation, and $\left[\mathrm{T}_{\mathrm{GC}}^{-1}\mathbf{x}\right](\mathbf{Z}) \simeq \mathbf{X}+\boldsymbol{\rho}_0$ and $\left[\mathrm{T}_{\mathrm{GC}}^{-1}\mathbf{v}\right](\mathbf{Z}) \simeq \dot{\mathbf{X}}+\dot{\boldsymbol{\rho}}_0$. Note that $\left[\mathrm{T}_{\mathrm{GC}}^{-1}\mathbf{x}\right](\mathbf{Z})$ does not denote the particle position in the gyro-center phase space and hence the **E**×**B** drift is not included in $\dot{\mathbf{X}}$. Electrostatic fluctuations are included in

$$\mathrm{T}_{\mathrm{Gy}}^{*}F = F + \{S_1, F\} + \cdots \quad (3)$$

where {,} denote the standard guiding-center Poisson brackets [7]. Usually only the lowest order term, $S_1^{(1)} = (e/\Omega)\int \tilde{\varphi}\, d\xi$, is considered in the pull-back transformation (3) where $\tilde{\varphi}$ denotes the gyro-phase dependent part of the electrostatic potential. However, the higher order solution for S_1

$$S_1^{(2)} = -\frac{1}{\Omega}\left(\frac{\partial}{\partial t} + U\mathbf{b}\cdot\boldsymbol{\nabla}\right)\int S_1^{(1)}\, d\xi \quad (4)$$

is needed to derive the polarization drift flux from the conventional representation (2) [8]. Recall that an equation determining S_1 is given by [6,9]

$$\frac{\partial S_1}{\partial t} + \{S_1, H_0\} = \tilde{\varphi} \quad (5)$$

with the guiding-center Hamiltonian $H_0 = \mu B + mU^2/2$ and it is solved order by order under the low frequency assumption. The time derivative term in $S_1^{(2)}$ yields the polarization drift flux. Belova used the conventional representation to derive an explicit representation of the perpendicular gyroviscous force from the gyrokinetic model [10]. In the derivation, to calculate FLR terms, not only $S_1^{(2)}$ but also S_2, the generating function at second order, must be considered in the pull-back transformation.

3. Pure push-forward representation

The other exact representation is given by

$$\boldsymbol{\Gamma}(\mathbf{r}) = \int d^6\mathbf{Z}\, \mathcal{J}\mathrm{T}_{\mathrm{Gy}}^{-1}\mathrm{T}_{\mathrm{GC}}^{-1}\mathbf{v}F\delta^3\left(\mathrm{T}_{\mathrm{Gy}}^{-1}\mathrm{T}_{\mathrm{GC}}^{-1}\mathbf{x}-\mathbf{r}\right) \quad (6)$$

where $\mathrm{T}_{\mathrm{Gy}}^{-1}\mathrm{T}_{\mathrm{GC}}^{-1}\mathbf{x} \simeq \mathbf{X}+\boldsymbol{\rho}_0+\boldsymbol{\rho}_1$ and $\mathrm{T}_{\mathrm{Gy}}^{-1}\mathrm{T}_{\mathrm{GC}}^{-1}\mathbf{v} \simeq \dot{\mathbf{X}}+\dot{\boldsymbol{\rho}}_0+\dot{\boldsymbol{\rho}}_1$ denote the particle position and velocity in the gyro-center space, respectively, and $\boldsymbol{\rho}_1 = \{\mathbf{X}+\boldsymbol{\rho}_0, S_1\}$ is the gyro-center displacement vector. This is the pure push-forward representation because the pull-back transformation of the distribution function does not appear. In the representation the effects of electrostatic fluctuations enter through $\boldsymbol{\rho}_1$ and $\dot{\mathbf{X}}$ mainly. It is noted that $\dot{\mathbf{X}}$ includes the **E**×**B** drift here. Using the lowest order solution $S_1^{(1)}$ and taking the long wavelength limit $\tilde{\varphi} \simeq \boldsymbol{\rho}_0\cdot\boldsymbol{\nabla}\varphi(\mathbf{X})$, we have

$$\boldsymbol{\rho}_1 = -\frac{1}{B\Omega}\boldsymbol{\nabla}_{\perp}\varphi + \frac{1}{eB}\mathbf{b}\times\boldsymbol{\nabla}S_1^{(1)}.$$

The polarization drift flux is obtained from $\dot{\boldsymbol{\rho}}_1$ directly. Thus the higher order term $S_1^{(2)}$ is not necessary for the representation (6).

4. Summary

We have compared two formally different push-forward representations of particle flux in the standard electrostatic gyrokinetic model by deriving the polarization flux from them. The difference between the two is the gyro-center transformation part which contains S_1, the scalar function generating the gyro-center transformation. A high order solution of S_1 including a time derivative term is needed to obtain the polarization flux if we use the conventional one in which the gyro-center part appears as the pull-back transformation of the distribution function. In contrast, the lowest order solution is sufficient for the other pure push-forward representation in which the time derivative of the gyro-center displacement vector gives the polarization flux directly. Furthermore, the particle flux representation with FLR terms that Belova calculated using the conventional representation and high order solutions of generating functions can be recovered using the other one with the lowest order solution only [11].

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