

Development of a new MHD equilibrium code
 新しいMHD平衡コードの開発

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For the analysis of the three-dimensional MHD equilibrium, it is very important to consider the effect of the magnetic islands or stochastic regions. In this research, we will develop the three-dimensional MHD equilibrium code in order to analyze that exactly. Here, we are developing the two-dimensional calculation code applied to an axisymmetric tokamak as the initial step.

1. Introduction

For the analysis of the three-dimensional MHD equilibrium, it is important to analyze the magnetic islands or stochastic regions. Several numerical codes like HINT and PIES have been applied for the analysis of the three-dimensional MHD equilibrium. The features of these codes are shown in table.1.

HINT	<ul style="list-style-type: none"> the cylindrical coordinate system is used in the latest version. MHD equilibrium equations are solved by the relaxation method.
PIES	<ul style="list-style-type: none"> the magnetic coordinate system is used to solve the parallel current. MHD equilibrium equations are solved by the iteration method. the magnetic field is obtained by Poisson's equation.

table.1 Code feature

Compared with these codes, the features of the code in this research are

- a) the cylindrical coordinate system is used.
- b) MHD equilibrium equations are solved by the iteration method.
- c) the magnetic field is obtained by Biot-Savart law.

The purpose of this research is to develop the three-dimensional calculation code. Here, we develop the two-dimensional calculation code applied to an axisymmetric tokamak as the initial step. In addition, the plasma boundary is specified by the limiter.

2. Research method

A simple description of a tokamak configuration is given by the MHD equations for force balance.

$$\nabla P = \vec{j} \times \vec{B} \tag{2.1}$$

$$\mu_0 \vec{j} = \nabla \times \vec{B} \tag{2.2}$$

$$\nabla \cdot \vec{B} = 0 \tag{2.3}$$

In our method, these equations are solved directly. The iterative procedure employed to solve the equations is illustrated in Fig.1. The role of each step is as follows.

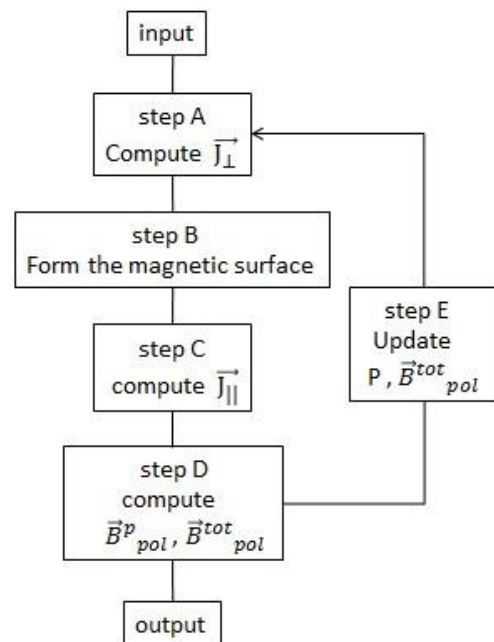


Fig.1 Code flow chart

<Step A>

The vector product of 'B' with Eq.(2.1) leads to an expression for the current perpendicular to the magnetic field line.

$$\vec{J}_{\perp} = \frac{\vec{B} \times \nabla p}{|\vec{B}|^2} \quad (2.4)$$

In this step, we calculate \vec{J}_{\perp} by using Eq.(2.4).

<Step B>

We solve the equation of the magnetic field line by using Runge-Kutta method. Furthermore, we obtain (R,Z) coordinate on magnetic surfaces as the function of (Ψ, θ) . Those are used in STEP C.

<Step C>

Taking the divergence of Eq.(2.2) yields

$$\nabla \cdot \vec{J} = 0 \quad (2.5)$$

From the Eq.(2.5)

$$\nabla \cdot \vec{J}_{\parallel} = - \nabla \cdot \vec{J}_{\perp} \quad (2.6)$$

is obtained obviously. By solving the magnetic differential equation, we can obtain the current parallel to the magnetic field line.

<Step D>

Plasma current is obtained by adding \vec{J}_{\perp} to \vec{J}_{\parallel} .

$$\vec{J}^p = \vec{J}_{\perp} + \vec{J}_{\parallel}$$

The magnetic field generated by the plasma current is calculated by Biot-Savart law. Here, we use the appropriate Green's function for a toroidal current source.

$$\Psi_{pol}^p = 2\pi\mu_0 \sum J^p G$$

The total magnetic field is obtained by the below process.

$$\begin{aligned} \Psi_{pol}^{total} &= \Psi_{pol}^p + \Psi_{pol}^{coil} \\ \Psi_{pol}^{total} &\rightarrow B_{pol}^{total} \text{ (differential)} \end{aligned}$$

<Step E>

Pressure distribution and magnetic field are updated in this process. These values are substituted into the calculation in StepA.

<Output>

Magnetic field, the shape of the magnetic surface, safety factor, and Pfirsch-Schlüter current are calculated.

3. Results

In this research, the initial input parameters are as follows. Plasma and computational domains are set as shown in Fig.2.

<Initial input parameters>

- Pressure distribution

$$P = 3.1 \times 10^6 (1 - \Psi)^2 [\text{Pa}]$$

- Current distribution

$$\sigma = \sigma_{00} (1 - \Psi)^2$$

- Total toroidal current

$$I_{total} = 4.8 \text{ MA}$$

- vacuum field

$$\vec{B}_{pol}^{coil} = 0.37 \vec{e}_z [\text{T}]$$

$$\vec{B}_{tor}^{coil} = \frac{55.72}{R} \vec{e}_t [\text{T}]$$

In these parameters, Ψ is the normalized toroidal flux. σ is $\langle \vec{J} \cdot \vec{B} \rangle / \langle B^2 \rangle$. These parameters are set so that the value of beta is 1% at the magnetic axis.

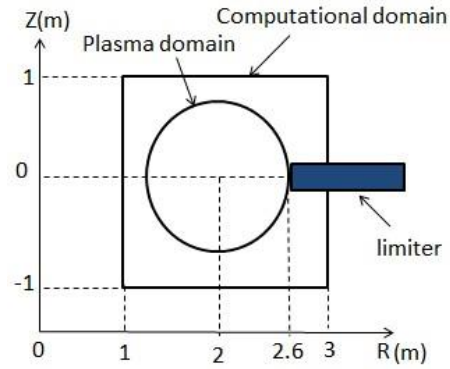


Fig.2 computational domain

Computational domain is a square[2m × 2m]. The limiter is located at R=2.6, Z=0.

This computational method is now under development. The magnetic field, safety factor, the shape of the magnetic surface, and Pfirsch-Schlüter current will be estimated in the conference.