Three-Dimensional Numerical Analysis of Diamagnetic Effects on Interchange Mode in Heliotron Plasmas

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In this paper, the influence of two-fluid diamagnetic flows on the interchange mode in the Large Helical Device (LHD) plasmas is investigated with the MIPS code [1]. The stabilization/destabilization of the linear growth rate of the interchange mode depends on the ratio between the diamagnetic frequency and the single fluid growth rate \( \gamma_t \). In the regime where \( \gamma_t < \omega_i \), the two-fluid effects are stabilizing, in agreement with the well-known theory. However when \( \gamma_t > \omega_i \), the two-fluid effects can be destabilizing.

1. Introduction

The magnetohydrodynamic (MHD) stability of heliotrons has not yet been fully understood. This stability against interchange modes, which are pressure driven modes, depends not only on the plasma \( \beta \) but also on the horizontal position of the vacuum magnetic axis \( R_{ax} \). Increasing \( R_{ax} \) makes the plasma more stable, however for large \( R_{ax} \) the confinement of high energy particles, which is crucial in a burning fusion plasma, is degraded. Thus a trade-off is required to obtain an optimum configuration with both good MHD stability and good particle confinement.

In LHD, the original value of \( R_{ax} \) is 3.75 m. It corresponds to the Mercier criterion [2] which gives the stability boundary against high toroidal wave number interchange modes, computed based on ideal MHD. Recently, experiments have shown that even for smaller values, down to \( R_{ax} \approx 3.6 \) m, the machine can be operated safely up to \( \beta \approx 5\% \) on the magnetic axis without major MHD event [3].

This means that LHD plasmas are more stable than predicted by ideal MHD. Understanding this stability is of major importance for the prediction of the characteristics of future heliontron fusion reactors. It is well known that plasma rotation is one of the factors which can improve the stability. Including toroidal rotation in a 3D equilibrium code is a difficult problem and is not addressed here. We carry out a study of the impact of the two-fluid diamagnetic flows on the interchange mode stability with the MIPS code. The MIPS code solves a model close to the two-fluid model of Hazeltine and Meiss. We first present the MHD model and the simulation conditions in section 2, and then describe and discuss the results in section 3. A conclusion follows in section 4.

2. MHD Model

The normalized equations for the plasma mass density \( \rho \), velocity \( \mathbf{v} \), pressure \( p \) and magnetic field \( \mathbf{B} \) are as follows:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \frac{\mathbf{J} \times \mathbf{B}}{\rho} + \frac{4}{3} \nabla \left[ \rho \nabla \cdot \mathbf{v} \right] - \nabla \times \left[ \rho \mathbf{v} \times \nabla \mathbf{v} \right]
\]

\[
\frac{\partial p}{\partial t} + \nabla \cdot (\rho \mathbf{v}) \left( \mathbf{v} \cdot \nabla \mathbf{v} / \rho \right) = \frac{1}{\rho} \frac{\partial \mathbf{B}^2}{\partial t} - \nabla \cdot (\rho \mathbf{v}) \nabla \mathbf{v}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \frac{\mu_0}{\rho} \mathbf{J}
\]

In these equations, the variable \( \mathbf{v} = \mathbf{E} \times \mathbf{B} / B^2 + \mathbf{v}_i \mathbf{B} / B \) represents the MHD velocity, and \( \mathbf{v}_i = \mathbf{B} \times \nabla p / (\rho B^2) \) is the normalized ion diamagnetic velocity. The normalization is as follows: the magnetic field is normalized to the field magnitude on the magnetic axis \( B_0 \), the mass density to the density on the magnetic axis \( \rho_0 \), the velocity to the Alfvén velocity \( V_A = B_0 / \sqrt{\mu_0 \rho_0} \) and the pressure to \( \rho_0 V_A^2 \). The time is normalized to the Alfvén time \( \tau_A = x_0 / V_A \), where \( x_0 \) is the distance normalization. In addition, \( D_\perp, \chi_\perp, \mathbf{v} \) and \( \eta \) are, respectively, the particle and heat diffusion coefficients, the viscosity and the resistivity, all being constant. As a result of this normalization, all the diamagnetic terms are multiplied by the normalized skin depth \( \delta_i = K / \sqrt{\mu_0} \), where \( K = m_i / (\mu_0 e^2) / x_0 \) is a constant (\( m_i \) is the ion mass and \( x_0 \) the distance normalization). Thus, fixing the diamagnetic parameter amounts to fixing the density. When the density varies between \( 10^{20} \) and \( 10^{20} \) m\(^{-3} \), \( \delta_i \) varies between 0.23 and 0.023.
3. Results

We carried out 2 sets of non-linear simulations in the inward-shifted configuration, for which the linear growth rate only is studied. The equilibrium is obtained with HINT2 code [4], which does not assume the existence of flux surfaces. The first set (i) has $\beta=2\%$, whereas the second (ii) has $\beta=1\%$. The single fluid growth rate of the $\beta=2\%$ case is approximately 20 times larger than the $\beta=1\%$ case, however the diamagnetic frequency $\omega_i^d = 2\pi m \Delta \phi / (\phi dl / |W_i^d|)$, where $m$ is the poloidal mode number, scales with $\beta$, so there is only a factor of 2 between the two cases. As a result, using values of the density typical of the experiment, $\omega_i^d$ is typically smaller than the single fluid growth rate $\gamma_i$ in case (i), and larger in case (ii). The results are compared with the expectation from simplified theory, which by replacing $\omega^2$ by $\omega(\omega - \omega_i^d)$ in the dispersion relation [5], gives

$$\gamma = \Re(\omega) = \gamma_i \sqrt{1 - (\omega_i^d / 2\gamma_i)^2}$$

(5)

$$\omega_i = \Im(\omega) = \omega_i^d / 2.$$ (6)

Fig. 1 and Fig. 2 show the results. The most unstable single fluid mode is $(m/n)=(4/3)$. In case (i), the stabilization and frequency approximately follow the theoretical expectation. The yellow point in Fig. 1 at the right end of the figure has a different mode number, suggesting full stabilizing of the 4/3 mode for $\omega_i^d \approx 2\gamma_i$. However, in case (ii), the result is totally different from theory, with a significant destabilization instead of stabilization. Furthermore, the mode rotates in electron direction rather than ion.

This indicates that the diamagnetic flows may be a poor candidate to explain the good stability of the experiment. Indeed, when the growth rate is large, a very low density is required to obtain a significant stabilization (less than 10% reduction of the growth rate for $n=10^{19}$ m$^{-3}$), and when it is small, the diamagnetic effects are destabilizing. This latter result is unexpected because diamagnetic effects almost always lead to stabilization, though a minor destabilization by electron diamagnetic term (not used here) has been reported in Ref. [6]. Thus this result must still be checked very carefully since they may be under resolved.

Fig. 1– Diamagnetic stabilization in case (i), $\beta=2\%$. The agreement with the theoretical expectation is reasonable.

Fig. 2– Diamagnetic destabilization in case (ii), $\beta=1\%$. The simulation and theoretical expectation are extremely different.

4. Conclusion

Our preliminary results indicate that in the inward-shifted configuration, the diamagnetic effects are either weakly stabilizing in the regime where $\gamma_i > \omega_i^d$, or destabilizing when $\gamma_i < \omega_i^d$, where $\omega_i^d$ is evaluated with values of the density relevant to the experiment. These results must be checked by resolution scans, and a more sophisticated dispersion relation, applicable to the present case, must be derived. This will be very helpful in the interpretation of the results.

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References