Momentum and energy input calculations for NBI heated plasmas by using an eigen function method and a non-linear collision operator

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In the momentum and energy balance analyses for multi-ion-species plasmas sustained by tangentially injected neutral beams, it is required to know how the injected parallel momentum and energy are distributed to each target particle species. In handling the parallel momentum, an important configuration effect is the fast ion trapping effect. By using the eigen function method, it can be included in the neoclassical transport calculations for the target plasma species in non-symmetric stellarator/heliotron devices. In this co-existence of the fast velocity moment of the fast ion velocity distribution function and the beam driven plasma flows, non-linear collision effects also may affect on the distribution of the injected energy. We discuss these physical processes in this presentation.

1. Introduction

Recently, parallel plasma flow velocities of NBI heated plasmas in Heliotron-J were successfully explained by the neoclassical transport theory [1]. That study applied a recently developed moment equation approach for general non-symmetric toroidal plasmas including the external momentum input [2]. In the moment method, problems including the field particle portion $C_{ab}(f_{abM}, f_{ab})$ of the linearized collision operator are converted to generalized parallel force balance expressed in an algebraic form. The recent study handled the external parallel momentum input by including the parallel friction collision moments $m_a \int \xi L_j^{(3/2)}(x_a^2) C_{ab}(f_{abM}, f_{ab}) d^3v$ of each target plasma species (denoted by the subscript “a”) with the fast ions (“f”) in this simultaneous algebraic equation. Here, $L_j^{(3/2)}(K) \equiv \left(e^{K} K^{-\alpha} / j! \right) d^j \left(e^{-K} K^{j+\alpha} \right) / dK^j$ is the Laguerre (Sonine) polynomial corresponding to the algebraic expression of the energy space structure, and $x_a^2 \equiv m_a v^2 / (2T_a)$. The fast ion birth profile was obtained by using the HFREYA and MCNBI, which are parts of a widely used NBI analysis code FIT3D [3]. Although the prompt orbit effect in non-symmetric toroidal configurations just after the beam ionization is taken into account in this method, a simple analytical formula of the fast ion velocity distribution $f_f$ for uniform magnetic field strength $\nabla B = 0$ is used for the collision integrals. It means that the fast ion trapping effect, which will be important for lower energy regions of $f_f$ broadened to full pitch angle range, is neglected. Therefore a more systematic method for the friction collision moments in general non-symmetric stellarator/heliotron configurations is required for more quantitative understandings of physical processes determining plasma flows. Here we shall apply the eigen function method, which is originally developed for axisymmetric tokamaks [4], for this kind of stellarator/heliotron studies.

2. Eigen function

This method is motivated by a fact that the neoclassical parallel damping force in the collisionless banana regime is determined only by $\left(1 - \lambda B / B_M\right)^{1/2}$ with $\lambda \equiv \mu B_M / \omega$ corresponding to the surface-averaged parallel drift velocity of circulating orbits. Therefore the theories for axisymmetric tokamaks can be extended to stellarator/heliotron configurations. The eigen functions $\Lambda_n(\lambda)$ is defined as solutions of following 1-D differential equation in the circulating pitch-angle range $0 \leq \lambda \leq 1$ with the eigen values $\kappa_n$.

$$\frac{\partial}{\partial \lambda} \lambda \left(1 - \lambda B / B_M\right)^{1/2} \partial \Lambda_n / \partial \lambda = \kappa_n \left(1 - \lambda B / B_M\right)^{1/2} \Lambda_n$$

$\Lambda_n(0) = 1, \Lambda_n(1) = 0$

The eigen values $\kappa_n$ satisfying the boundary conditions $\Lambda_n(0) = 1, \Lambda_n(1) = 0$ are solved numerically with the moment equation approach for general non-symmetric stellarator/heliotron configurations by using the FIT3D code [3].
configurations will be shown in the presentation. Numerical examples in LHD and Heliotron eigene Legendre transport calculations target ions, is easily included reduces the substantial parallel momentum input for Therefore, the fast ion trapping effect, which expression of the spherical coordinate general Laguerre orders As shown in Ref.[2], the arbitrary function velocity expansion by $A_n(\lambda)$, and the surface-averaged velocity-space integrals $\left< B \int F(v) \xi f d^3v \right>$ with arbitrary function $F(v)$, for which only the fast ions in $0 \leq \lambda \leq 1$ contribute, is given by

$$\frac{1}{\mathcal{V}} \left< B \int F(v) \xi f d^3v \right> = \frac{B^2}{2B_M} \int_0^{V_b} \frac{F(v)}{v^2 v_T e^3} \left( 3 \sqrt{\pi/2} G(x_c) + v^2 \right) \times \left[ \sum_{n=1}^{6} \int_0^1 \frac{1}{\lambda} \left< \frac{\partial}{\partial \lambda} \left( 1 - \lambda B/B_M \right)^{1/2} A_n^2 d\lambda \right> \right] \times \left[ \frac{1}{\lambda} \left< \frac{V(v)}{V(v_b)} \right> \left( \frac{\partial}{\partial \lambda} \right)^{1/3} \right] \left[ \frac{\partial}{\partial \lambda} \left( 1 - \lambda B/B_M \right)^{1/2} \right] \frac{2\pi v^2 d\lambda}{V^2}$$

As shown in Ref.[2], the $m_a \left< v \xi L_j^{1/2} \left( x_n^2 \right) C_{ab} \left( f_{aM} \cdot f_l d^3v \right) \right>$ moments of general Laguerre orders $j$, which are obtained by partial integrals of the spherical coordinate expression of $C_{ab} \left( f_{aM} \cdot f_l \right) \ [5]$, also have this form. Therefore, the fast ion trapping effect, which reduces the substantial parallel momentum input for target ions, is easily included in the neoclassical transport calculations by replacing the Legendre-expanded $f_l \ [2]$ by the expansion by this eigen function.

Numerical examples in LHD and Heliotron-J configurations will be shown in the presentation.

References