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Momentum and energy input calculations for NBI heated plasmas by using an eigen function method and a non-linear collision operator
固有関数法と非線形衝突項を用いたNBI加熱プラズマの運動量・エネルギー入力解析方法

Shin Nishimura[1], Yuji Nakamura[2] and Kenji Nishioka[2]
西村伸[1]，中村祐司，西岡賢二[2]

[1]*National Institute for Fusion Science, Oroshi-cho 322-6, Toki 509-5292, Japan*
[2]*Graduate School of Energy Science, Kyoto University, Gokasho, Uji 611-0011, Japan*
[1]核融合科学研究所　〒509-5292　岐阜県土岐市下石町322-6
[2]京都大学大学院エネルギー科学研究科　〒611-0011 京都府宇治市五ケ庄

In the momentum and energy balance analyses for multi-ion-species plasmas sustained by tangentially injected neutral beams, it is required to know how the injected parallel momentum and energy are distributed to each target particle species. In handling the parallel momentum, an important configuration effect is the fast ion trapping effect. By using the eigen function method, it can be included in the neoclassical transport calculations for the target plasma species in non-symmetric stellarator/heliotron devices. In this co-existence of the fast velocity moment of the fast ion velocity distribution function and the beam driven plasma flows, non-linear collision effects also may affect on the distribution of the injected energy. We discuss these physical processes in this presentation.

1. Introduction
Recently, parallel plasma flow velocities of NBI heated plasmas in Heliotron-J were successfully explained by the neoclassical transport theory [1]. That study applied a recently developed moment equation approach for general non-symmetric toroidal plasmas including the external momentum input [2]. In the moment method, problems including the field particle portion $C_{ab}\left(\langle f_{a\mathrm{M}}\rangle, f_{b1}\right)$ of the linearized collision operator are converted to generalized parallel force balance expressed in an algebraic form. The recent study handled the external parallel momentum input by including the parallel friction collision moments $m_a\int v\xi L_j^{(3/2)}\left(x_a^2\right)C_{ab}\left(f_{a\mathrm{M}}, f_{\mathrm{f}}\right)\mathrm{d}^3\mathbf{v}$ of each target plasma species (denoted by the subscript "a") with the fast ions ("f") in this simultaneous algebraic equation. Here, $L_j^{(\alpha)}(K)\equiv\left(e^K K^{-\alpha}/j!\right)\mathrm{d}^j\left(e^{-K}K^{j+\alpha}\right)/\mathrm{d}K^j$ is the Laguerre (Sonine) polynomial corresponding to the algebraic expression of the energy space structure, and $x_a^2\equiv m_a v^2/\left(2\langle T_a\rangle\right)$. The fast ion birth profile was obtained by using the HFREYA and MCNBI, which are parts of a widely used NBI analysis code FIT3D [3]. Although the prompt orbit effect in non-symmetric toroidal configurations just after the beam ionization is taken into account in this method, a simple analytical formula of the fast ion velocity distribution f_{f} for uniform magnetic field strength ∇B=0 is used for the collision integrals. It means that the fast ion trapping effect, which will be important for lower energy regions of f_{f} broadened to full pitch angle range, is neglected. Therefore a more systematic method for the friction collision moments in general non-symmetric stellarator/heliotron configurations is required for more quantitative understandings of physical processes determining plasma flows. Here we shall apply the eigen function method, which is originally developed for axisymmetric tokamaks [4], for this kind of stellarator/heliotron studies.

2. Eigen function
This method is motivated by a fact that the neoclassical parallel damping force in the collisionless banana regime is determined only by $\left\langle(1-\lambda B/B_{\mathrm{M}})^{1/2}\right\rangle$ with $\lambda\equiv\mu B_{\mathrm{M}}/w$ corresponding to the surface-averaged parallel drift velocity of circulating orbits. Therefore the theories for axisymmetric tokamaks can be extended to stellarator/heliotron configurations. The eigen functions $\Lambda_n(\lambda)$ is defined as solutions of following 1-D differential equation in the circulating pitch-angle range $0\le\lambda\le 1$ with the eigen values κ_n.

$$\frac{\partial}{\partial\lambda}\lambda\left\langle(1-\lambda B/B_{\mathrm{M}})^{1/2}\right\rangle\frac{\partial\Lambda_n}{\partial\lambda}=\kappa_n\frac{\partial\left\langle(1-\lambda B/B_{\mathrm{M}})^{1/2}\right\rangle}{\partial\lambda}\Lambda_n$$

$$\Lambda_n(0)=1,\ \Lambda_n(1)=0$$

The eigen values κ_n satisfying the boundary condition at λ=0,1 are found by the shooting method. Figure 1 shows an example obtained in a (L,N)=(2,10) heliotron configuration. These functions satisfying following orthogonal relation between different eigen values $m\neq n$.

$$\left\langle \int_0^1 \frac{\partial (1-\lambda B/B_{\mathrm{M}})^{1/2}}{\partial \lambda} \Lambda_m(\lambda)\Lambda_n(\lambda)\mathrm{d}\lambda \right\rangle = 0$$

Then the general functions $H(v,\lambda)$ satisfying $\mathbf{b}\bullet\nabla_{\lambda=\mathrm{const}}H=0$ in the circulating pitch-angle $0\le\lambda\le 1$ can be expressed by the orthogonal expansion by $\Lambda_n(\lambda)$, and the surface-averaged velocity-space integrals $\left\langle B\int F(v)\xi f_{\mathrm{f}}\mathrm{d}^3\mathbf{v}\right\rangle$ with arbitrary function $F(v)$, for which only the fast ions in $0\le\lambda\le 1$ contribute, is given by

$$\frac{1}{\tau_{\mathrm{S}}}\left\langle B\int F(v)\xi f_{\mathrm{f}}\mathrm{d}^3\mathbf{v}\right\rangle$$

$$= -\frac{\left\langle B^2\right\rangle}{2B_{\mathrm{M}}}\int_0^{v_{\mathrm{b}}}\frac{F(v)}{v^2 v_{\mathrm{Te}}(3\sqrt{\pi}/2)G(x_{\mathrm{e}})+v_{\mathrm{c}}^3}\times$$

$$\left[\sum_{n=1}^{6}\frac{\left\langle \int_{-1}^{1}\sigma\Lambda_n(\lambda)S_0(\theta,\zeta,\xi)\mathrm{d}\xi\right\rangle}{\left\langle \int_0^1 \frac{\partial(1-\lambda B/B_{\mathrm{M}})^{1/2}}{\partial\lambda}\Lambda_n^{\,2}\,\mathrm{d}\lambda\right\rangle}\times \right.$$

$$\left. \int_0^1 \Lambda_n(\lambda)\mathrm{d}\lambda\left\{\frac{V(v)}{V(v_{\mathrm{b}})}\right\}^{\kappa_n Z_2/3}\right]2\pi v^2 \mathrm{d}v$$

As shown in Ref.[2], the $m_a\left\langle\int v\xi L_j^{(3/2)}\left(x_a^2\right)C_{ab}\left(f_{a\mathrm{M}},f_{\mathrm{f}}\right)\mathrm{d}^3\mathbf{v}\right\rangle$ moments of general Laguerre orders j, which are obtained by partial integrals of the spherical coordinate expression of $C_{ab}\left(f_{a\mathrm{M}},f_{\mathrm{f}}\right)$ [5], also have this form. Therefore, the fast ion trapping effect, which reduces the substantial parallel momentum input for target ions, is easily included in the neoclassical transport calculations by replacing the Legendre-expanded f_{f} [2] by the expansion by this eigen function.

Numerical examples in LHD and Heloitron-J configurations will be shown in the presentation.

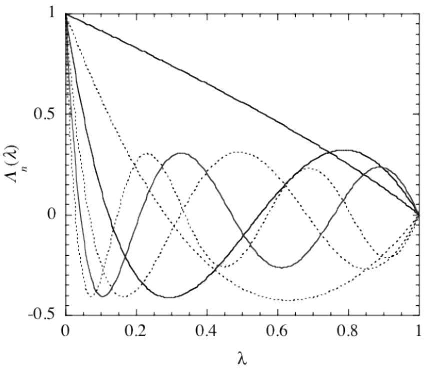

Fig.1 An example of $\Lambda_n(\lambda)$ for $\int_0^1\lambda\left\langle(1-\lambda B/B_{\mathrm{M}})^{1/2}\right\rangle^{-1}\mathrm{d}\lambda = 0.821$

References

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