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A new solution method for longstanding problems of resistive MHD stability analysis
抵抗性 MHD 安定性解析の未解決問題に対する新しい近似解法

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We have developed a new solution method for longstanding problems of resistive magnetohydrodynamics (MHD) stability analysis. Since this is a conference proceeding of an invited talk, the main body of the study has been published elsewhere. In this proceeding, we point out the difficulties of the classical method based on the matched asymptotic expansion, and briefly explain notion of our new method.

1. Introduction

One of the important pieces in nuclear fusion development is magnetohydrodynamics (MHD) stability analysis of confined plasmas. Finite plasma resistivity introduces several resistive MHD instabilities such as tearing modes. Since large-scale magnetic islands degrade the plasma confinement significantly and even cause disruption, control and suppression of the islands are one of the urgent issues. To understand physics of them, accurate calculation of the resistive MHD stability is indispensable.

A standard method of the stability analysis has been the asymptotic matching method[1,2]. The method relies on assumptions such as smallness of the resistivity and the slow dynamics. Then the resistivity and the inertia terms can be neglected in most of the region in the plasma, that we call outer region. There we solve the inertia-less, ideal MHD equation or the so-called Newcomb equation[3]. In the outer region, the dominant effect is magnetic tension. The magnetic tension however vanishes at a resonant surface inside the plasma if it exists, and thus resistivity and the inertia need to be retained in a thin layer around the resonant surface, which we call an inner layer. Since the inner layer is thin, we can simplify the governing equation. To focus on the inside of the thin layer and on the slow dynamics, we may re-scale the radial coordinate and the frequency by using resistivity as a small parameter, leading to the inner equation.

The resistivity term has the highest-order spacial derivative; it is singular perturbation. Since that term is dropped in the outer region, the number of independent solutions becomes two, instead of four in the inner layer. Two of the four independent solutions in the inner layer have the same asymptotic forms which can be matched onto the outer solution. The two solutions can be obtained as the Frobenius series around the resonant surface, and are called small and large solutions. The ratio of the small solution to the large solution, called matching data, plays crucial role in the asymptotic matching method. The matching then gives us the dispersion relation.

The asymptotic matching method is well established mathematically. However, it has some difficulties in practice. (i) Although the resistivity and the inertia are neglected in the outer region, they can be important there if the plasma is close to marginal stability against ideal MHD. A plasma close to its ideal MHD stability limit may be preferable to improve efficiency of a fusion reactor. Such a situation may be simulated by a cylindrical plasma with $q = 1$ surface inside the plasma, where q is the safety factor. The $m/n = 1/1$ internal kink mode is then marginally stable against ideal MHD, where m and n are the poloidal and toroidal mode numbers, respectively. (ii) The method cannot be applied in the first place if the resonant surface becomes an irregular singular point of the Newcomb equation. An important type of discharge in fusion development has non-monotonic q profile, and the minimum-q position can be such irregular singularity. (iii) Accurate numerical computation of the matching data is still difficult in toroidal plasmas even though some sophisticated theory have been developed[4,5]. (iv) In the plasma close to marginal stability against ideal MHD, the matching data diverges. Although a numerical scheme to calculate huge matching data was developed[6], it is reported that the accuracy of the matching data strongly depend on the local equilibrium accuracy and grid arrangement[7]. (v) Careful treatment is required in solving the inner equation numerically since the radial coordinate is re-scaled into unbounded space[8].

We pointed out these difficulties, and resolved them by our new matching method in Ref. [9,10]. The key ingredients of the new method are an utilization of an inner region with a *finite width*, and an ordering scheme for the outer region. We will briefly explain notion of the new method below. Readers interested in details of the formulation and applications may refer to Ref. [9,10].

2. Notion of new matching method

In our new method, we adopt a finite-width inner region, instead of infinitely thin inner layer. Neither the radial coordinate nor the frequency are re-scaled via small resistivity. The solutions in the inner and outer regions are then matched *directly, not asymptotically*, by imposing continuity of perturbed magnetic field at the boundaries, which are reasonably apart from the resonant surface. Since the boundaries or the matching points are apart from the singularity, we can fully avoid the difficulties in the numerical computation. Accurate computation of the matching data, including the divergent case, and the careful treatment of the unbounded space are unnecessary. Furthermore, our method is applicable to the irregular singularity case, since the method does not rely on the Frobenius series solution.

Note that the finite-width inner region was originally introduced for ideal MHD modes[11], and extended to resistive wall modes in rotating cylindrical plasmas[12]. The important difference from these studies is that the resistivity term increases the order of spacial derivative. We then need to select two of the four independent solutions in the inner region, that can match onto the outer solution. A natural idea for the selection may be to impose smooth disappearance of parallel electric field $E_\parallel$ as approaching the matching points from the inner side[9]. The application results were mostly satisfactory.

What was exceptional is the $m/n = 1/1$ internal kink mode. This mode is marginally stable against ideal MHD. In the $q < 1$ region, which is treated as ideal MHD region in the asymptotic matching method, $E_\parallel$ does not disappear smoothly. The remaining $E_\parallel$ indicate the relative importance of the resistivity and the inertia. Thus we have developed an ordering scheme for the outer region to include those effects perturbatively. The ordering is similar to the one adopted for the inner layer in the asymptotic matching method, but is different in the estimation of radial derivative. Since the boundary of the outer region is reasonably far from the singularity, we can assume that the radial derivative is of order unity. The ordering scheme then gives us a hierarchy of generalized Newcomb equations. The lowest- and the first-order equations agree with the conventional one, and the second-order equation includes the resistivity and the inertia as inhomogeneous terms. Note that the homogeneous part has the same differential operator at each order. By including the second-order correction, we have achieved sufficiently accurate stability calculations also for the concerned $m/n = 1/1$ internal kink modes.

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