

Gyrokinetic Analysis of Phase-Space Interactions in Two-Dimensional Electrostatic Turbulence

2次元静電乱流の位相空間相互作用に関するジャイロ運動論解析

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Various local and nonlocal interactions may take place in the phase-space turbulence of magnetized weakly collisional plasmas. Using electrostatic gyrokinetic simulations, it is shown that those interactions may be characterized by the ratio between two collisionless invariants in two dimensions. Theoretically such a variety of interactions may be explained by generalizing the argument of Fjørtoft [Tellus **5** (1953) 225]. Practical application might include the control of large-scale fluctuations by means of the excitation of a particular small-scale perturbation due to the nonlocal interactions.

1. Introduction

Plasmas in fusion device, solar wind and others are usually collisionless or at most weakly collisional. In such weakly collisional plasmas, turbulence proceeds in phase space [1-8]. In magnetized plasmas, free streaming of particles parallel to the ambient field line brings about linear phase mixing, which ends up with the collisional dissipation of entropy through creation of small-scale structures in parallel velocity [2].

On the other hand, phase mixing perpendicular to the ambient field proceeds nonlinearly due to the small-scale fluctuations of electrostatic potential [3]. When gyrokinetic (GK) theory is applied, the gyro-averaged potential differs among particles with different orbits, thus introducing their decorrelation. In this case the velocity and position spaces are strongly coupled, and as the turbulence proceeds, the entropy cascades in the velocity space as well as in the position space [4-8].

In two dimensions (2D) there is another collisionless invariant in addition to the entropy, which is associated with the electrostatic potential [6,8,9]. As the 2D Navier-Stokes (NS) turbulence exhibits a dual cascade [10,11], 2D GK also exhibits a dual cascade, where the difference from the NS turbulence is that both velocity and position spaces (or a phase space) need to be taken into account [9].

In this presentation, we show that there are various routes of interactions in the phase-space

turbulence [9]. The possibility of the nonlocal interactions may be a potential problem in constructing a renormalized theory of turbulence. We first introduce our model in Sec. 2 and present the numerical results in Sec. 3.

2. Model

We use the normalized 2D electrostatic GK equation

$$\frac{\partial g}{\partial t} + \left\{ \langle \phi \rangle_R, g \right\} = \langle C \rangle_R, \quad (1)$$

where g is the gyro-average of the perturbed distribution function, ϕ is the electrostatic potential, $\langle \bullet \rangle_R$ is the gyro-average with the gyro-center position R fixed, $\{a, b\} = \hat{z} \times \nabla a \cdot \nabla b$ is the Poisson bracket, and C is the collision operator. The electrostatic potential ϕ is obtained via the quasi-neutrality condition

$$\int \langle g \rangle_r dv = \phi - \langle \langle \phi \rangle_R \rangle_r, \quad (2)$$

where $\langle \bullet \rangle_r$ is the gyro-average with the particle position r fixed. Here we assumed zero response electrons where electrons make no contribution to the potential. This system of equations possesses two collisionless invariants

$$W = \frac{1}{2V} \iint g^2 dR dv, \\ E = \frac{1}{2V} \int \phi^2 + \phi \langle \langle \phi \rangle_R \rangle_r dr,$$

where W tends to cascade forward and E does

inversely. Equations (1)-(2) are numerically solved using an open-source, MPI-parallelized Fortran 95 code AstroGK [12].

3. Results

In order to characterize the velocity space structure we introduce the Hankel transform in the perpendicular velocity $\hat{g}(p) = \int J_0(pv_\perp)g(v_\perp)v_\perp dv_\perp$, where J_0 is the Bessel function. Position space structure may be characterized by the wave number k via the standard Fourier transform.

Figure 1 shows the spectral density distribution $W(k, p) = kp|\hat{g}(k, p)|^2$ for three runs of the decaying-turbulence simulations made with varying initial ratio between W and E ($\kappa = W/E$). While W corresponds to the spectral density over the whole (k, p) plane, E is concentrated only on the diagonal ($k = p$) as only those components contribute the electrostatic potential. Thus small κ corresponds to a state where spectral density is concentrated along the diagonal, while large κ corresponds to the other.

In the middle case [moderate κ ; Fig. 1(c) and (d)], small-scale initial condition exhibits a typical dual cascade where W cascades forward in a diffuse way and E cascades inversely along the diagonal. When κ/k_0^2 is smaller than unity [Fig. 1(a) and (b)], where k_0 is the wave number of the initial condition, the inverse cascade becomes non-local, which may be explained by the Kelvin-Helmholtz instability of the initial condition. When κ/k_0^2 is much larger than unity [Fig. 1(e) and (f)], on the other hand, the large-scale diagonal component is transferred to small scale directly as the spectral diffusion of W reaches the diagonal. This nonlocal transfer of E arises due to its conservation property during the evolution. Interestingly, in Fig. 1(f), the large-scale E is not sufficient to support the spectral diffusion of W and thus the spectral density contains a dip along the diagonal [Fig. 1(f)].

In the presentation, we will express our results in more detail including the theoretical explanation in conjunction with the Fjørtoft's theory [11].

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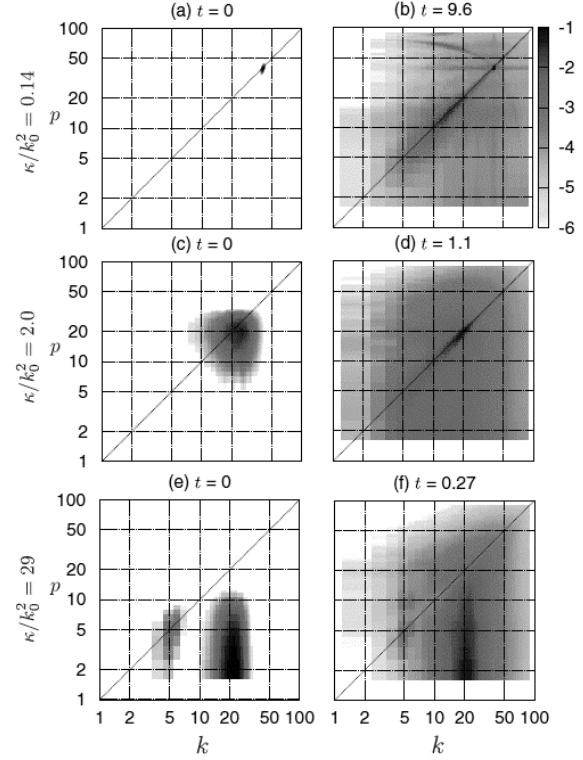


Figure 1: Example spectral distributions. Diagonal ($k = p$) is indicated by the dotted line.

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