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Shape Reconstruction Method Based on Magnetic Measurement and Boundary Element Method for the Peripheral Region in Non-Axisymmetric Fusion Plasma

磁気計測と境界要素法を用いた非軸対称核融合プラズマの周辺磁場領域形状の同定

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The Cauchy condition surface (CCS) method, originally developed for axisymmetric tokamak plasma, has been expanded to reconstruct the 3-D magnetic field profile outside the non-axisymmetric plasma in the Large Helical Device. The boundary integral equations (BIEs) in terms of 3-D vector potential for field sensors, flux loops and points along the CCS are solved simultaneously. With the use of 'twisted CCS', the reconstructed magnetic field shows acceptable accuracy. The magnetic field line tracing using the reconstructed field indicates the outer surface of the stochastic region precisely, and the last closed magnetic surface agrees fairly well with the reference one.

1. Introduction

Usually the plasma boundary shape in a nuclear fusion device is indirectly estimated with the aid of computing from signals of magnetic sensors located outside the plasma. Kurihara [1] proposed the Cauchy condition surface (CCS) method [1,2]. The method focuses on tokamaks, i.e., axisymmetric plasmas, so that the analyses can be made in a 2-dimensional (2-D), r-z system. On the other hand, 3-D analyses are required for non-axisymmetric plasma. In a helical-type device such as the Large Helical Device (LHD), it is important to consider the following characteristics of the plasma current:

(i) The plasma current itself is much weaker than the toroidal current in a tokamak device.

(ii) The dominant plasma current is the so-called Pfirsch-Schülter current, the average of which over a magnetic surface is zero. However, this current still has a 3-D profile.

The present work is an extension of the CCS method to non-axisymmetric, 3-D fusion plasma. This 3-D analysis consumes a huge number of unknowns, and then the problem becomes very ill-conditioned.

2. Three-Dimensional CCS Method

The Cauchy condition surface (CCS), where both the Dirichlet and the Neumann conditions are unknown, is hypothetically placed in a domain that can be supposed to be inside the plasma. In the analysis, no plasma current is assumed outside this CCS, where in reality plasma current does exist.

A 3-D Cartesian coordinate system is adopted to realize a boundary-only integral formulation. The first step of the analysis is to solve the following boundary integral equations (BIEs) and obtain the values of the vector potential and its derivative on the CCS in such a way that they will be consistent with the sensor signals [3].

(i) For magnetic field sensor locations i:

$$B_j - W_j^{(B)} = \sum_k \int_{\Gamma_{CCS}} \left\{ \left(L_k^j \phi_i^* \right) \frac{\partial A_k}{\partial n} - A_k \left(L_k^j \frac{\partial \phi_i^*}{\partial n} \right) \right\} d\Gamma. \quad (1)$$

$$(j = r, \varphi, z; \quad k = x, y, z)$$

The operator L_k^j corresponds to a term in $\mathbf{B} = \nabla \times \mathbf{A}$ with $\mathbf{A}$ expressed in Cartesian coordinates, $W_j^{(B)}$ is the contribution of external coil currents, and ϕ_i^* the fundamental solution of the Laplace equation.

(ii) For flux loops:

For example, the BIE for a loop in the toroidal direction is written as

$$\psi^{(Tor)} - W^{(Tor)} = \sum_k \int_{\Gamma_{CCS}} \left\{ \frac{\partial A_k}{\partial n} \left(\int_0^{2\pi} \eta_k \phi_i^* d\varphi \right) - A_k \left(\int_0^{2\pi} \eta_k \frac{\partial \phi_i^*}{\partial n} d\varphi \right) \right\} d\Gamma \quad (2)$$

$$(k = x, y)$$

with $\eta_x = -R \sin\varphi$ and $\eta_y = R\cos\varphi$.

(iii) For points i on the CCS (Γ_{CCS}):

$$\frac{1}{2} A_{k,i} = \int_{\Gamma_{CCS}} \left(\phi_i^* \frac{\partial A_k}{\partial n} - A_k \frac{\partial \phi_i^*}{\partial n} \right) d\Gamma. \quad (k = x, y, z) \quad (3)$$

The above three types of BIEs are discretized, coupled and expressed in a matrix equation form. Considering the 10-fold rotational symmetry of the

LHD in the toroidal direction, the number of unknowns is reduced by a factor of 10.

The matrix equation to be solved has the form

$$\mathbf{Dp} = \mathbf{g}, \tag{4}$$

where the solution vector $\mathbf{p}$ contains the vector potentials and their normal derivatives on the CCS. This equation is solved using the truncated singular value decomposition (TSVD) technique. The matrix $\mathbf{D}$ is decomposed as $\mathbf{D} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^{\mathrm{T}}$, where $\mathbf{U}$ and $\mathbf{V}^{\mathrm{T}}$ are orthogonal matrices and $\mathbf{\Lambda}$ is a diagonal matrix with positive singular value or zero components.

The regularized solution is given by

$$\mathbf{p} = \mathbf{V}\mathbf{\Lambda}_k^{-1}\mathbf{U}^{\mathrm{T}}\mathbf{g}. \tag{5}$$

Here $\mathbf{\Lambda}_k$ means that the singular values smaller than λ_k in $\mathbf{\Lambda}$ are omitted so that the condition number is not larger than 10^5.

Once all the components in $\mathbf{p}$ are known, the magnetic fields for arbitrary points can be calculated using Eq.(1) again.

3. Use of Twisted CCS

An axisymmetric CCS having a circular cross section is the simplest model [3]. However, the better idea is a 'twisted CCS' whose elliptic cross section rotates with the variation in vacuum vessel geometry in the toroidal direction. Independent of the toroidal angle, this CCS can keep a certain distance from its surface to the plasma boundary, so that a reduction in the numerical error can be expected. The CCS is divided into 48 boundary elements, each of which has 9 nodal points.

4. Numerical Examples

One here considers the plasma with a volume-averaged beta being $\langle\beta\rangle = 2.7\%$ in the LHD. The reference field and the sensor signals (126 flux loops and 440 field sensors) for this condition had been calculated beforehand using the HINT2 code [4].

In the greater part of the region outside the LCMS (last closed magnetic surface), the absolute errors of B_r, B_φ and B_z reconstructed using the 3-D CCS method were less than 0.01T.

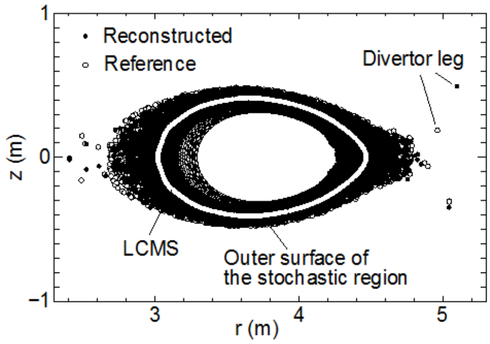

Fig.1. Field line tracing (1): Peripheral region

Magnetic field line tracing was then carried out. Figure 1 shows the Poincaré plots of the field line on the *r-z* plane at $\varphi = 18°$ (the horizontal elongated cross section). The white and the black round symbols are the results following the reference field and the reconstructed field, respectively. The white closed line indicates the LCMS given from the reference field. The reconstructed outer surface of the stochastic region shows a good agreement with the reference one.

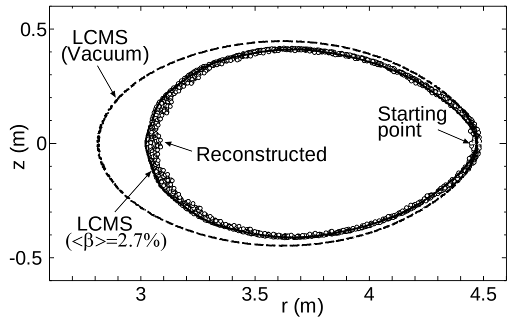

Fig.2. Field line tracing (2): LCMS

In Fig. 2, the dashed closed line shows the LCMS for the vacuum field, i.e., $\langle\beta\rangle = 0\%$. This is shifted outward when the beta takes the nonzero value, $\langle\beta\rangle = 2.7\%$, i.e., the reference LCMS in this case is the solid closed line. The round symbols show the results of the trace originating at the same starting point as the reference LCMS for $\langle\beta\rangle = 2.7\%$, but based on the reconstructed field obtained using the 3-D CCS method. They do not form a sharp closed surface, however, the round symbols are distributed almost along the reference LCMS for $\langle\beta\rangle = 2.7\%$.

5. Conclusion

A prototype of 3-D CCS method code has been developed, in which the formulation is based on the 3-D distribution of vector potential. A 3-D test calculation has been made for non-axisymmetric plasma in the LHD. The magnetic field outside the plasma can be reconstructed with a fairly acceptable accuracy if a large number of magnetic sensors can be located outside the plasma. The magnetic field line tracing indicates the plasma boundary precisely, and the LCMS agrees fairly well with the reference one. It should be stressed that they were reconstructed using only sensor signals.

References

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