24P166-B

The Theoretical study of Radiation Reaction under Ultrarelativistic laser
超相対論レーザーの下での放射の反作用の理論研究

Keita Seto, Hideo Nagatomo, Kunioki Mima
瀬戸慧大，長友英夫，三間圀興

Institute of Laser Engineering, Osaka University, 2-6 Yamada-oka, Suita, Osaka 565-0871, Japan
大阪大学・レーザーエネルギー学研究センター 〒565-0871 大阪府吹田市山田丘2-6

In Ultrarelativistic laser-plasma interaction over the intensity of 10^{22}W/cm^2, it is known the 'radiation reaction' effect becomes large because of the significant bremsstrahlung. If the laser intensity can become higher and higher, we consider, for example, the electron-proton pair creation appears. Therefore, the next of laser-plasma interaction field is the quantum domain. In this presentation, We show the quantum shift of the radiation reaction dynamics from the classical description, without electron-proton pair creations as the basic study of the next generation laser-plasma interactions.

1. Introduction

With the advent of ultra–relativistic laser, it will reach to the Intensity 10^{22} W/cm^2. If an electron is under this laser when laser intensity is larger than 10^{18}W/cm^2, an electron will carry out relativistic behavior. The most important phenomenon in this regime is an effect of ponderomotive force, and an electrons are pushed out in the propagation direction of laser. If we can get the laser intensity 10^{22} W/cm^2, the strong bremsstrahlung might be caused. Moreover, simultaneously with it, "radiation reaction force (or damping force)" works to charge particles. We could reach to describe this reaction force working on an electron in Minkowski spacetime, it is as follows [1]:

$$f_{\text{reaction}}{}^{\mu} = m_0\tau_0 \frac{d^2 w^\mu}{d\tau^2} + \frac{m_0\tau_0}{c^2} g\left(\frac{dw}{d\tau},\frac{dw}{d\tau}\right) w^\mu - m_0\tau_0 \frac{w^0 w^\mu - c^2\delta_0{}^\mu}{\left(w^0\right)^2 - c^2}\frac{d^2 w^0}{d\tau^2} \quad (1)$$

Where $\tau_0 = Q^2/6\pi\varepsilon_0 m_0 c^3$, w, τ and g is the relativistic 4-velocity, proper time and Lorentz metric. Before this equation appears, the Landau-Lifshitz (L-L) equation [2] as the method of perturbation of Lorentz-Abraham-Dirac (LAD) equation [3] is used. But, the L-L equation breaks the relation of relativistic covariance. Our Eq.(1) is satisfied with the covariance.

2. Quantum model of radiation reaction

In the next laser intensity generation (over 10^{24}W/cm^2), we consider the electron-positron pair creations and something effects appear [4]. This intensity level requires us to treat phenomena as QED (Quantum Electro Dynamics). However, we can consider an electron as quantum description under 10^{24}W/cm^2, without the pair creations. In this presentation, we suggest the quantum model of the radiation reaction without the electron-positron pair creation using the Dirac equation.

$$\left[i\hbar\gamma^\mu\left(\partial_\mu - \frac{ie}{\hbar}A_\mu\right) - mc\mathbb{I}\right]\psi = 0 \quad (2)$$

Here, $\gamma^{\mu=0,1,2,3}$ are Dirac matrix and A_μ means the electronic field, $A_\mu = (\phi/c, -\mathbf{A})$. In QED, we consider this field contains not only laser, but the bremsstrahlung from the electron. Under the theory of quantum field, the wave function ψ and the electromagnetic field A are described the fields operator. When the free electron's propagator is $S_{\text{free}}(x,x')$ and the solution of the Dirac equation of free electron is ψ_0, the full solution of Eq.(2) becomes

$$\psi(x) = \psi_0(x) - \int dx' S_{\text{free}}(x,x') e\gamma^\mu A_\mu(x'). \quad (3)$$

Therefore, the second term means photon-electron interaction. R. P. Feynman suggests that this is calculated by the Feynman diagram (see Fig.1) [5,6]. This diagram leads the probability (called the S-matrix) which the electron p and the laser $\hbar k$

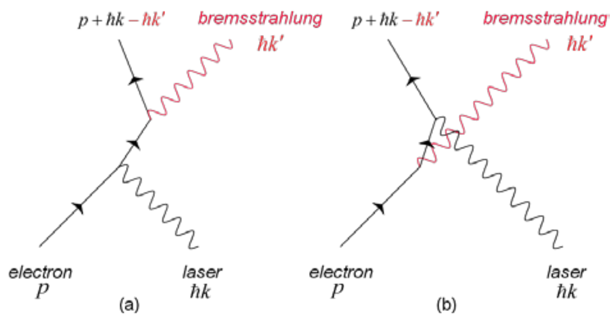

Fig.1 The Feynman diagram of laser-electron interaction

become $p+k-k'$ and the radiation k' (in the easiest perturbation). But, in QED, this calculation requires us to decide the condition of the initial and the final. It is described in the Fock space,

$$\mathcal{G}_{p,\hbar k}^{p+\hbar k-\hbar k',\hbar k'} = \left\langle p+\hbar k-\hbar k' \right| \otimes \left\langle \hbar k' \right| \left[-\int dx' S_{\text{free}}\left(x,x'\right) e\gamma^{\mu} A_{\mu}\left(x'\right) \right] \left| p \right\rangle \otimes \left| k \right\rangle . \quad (4)$$

Here, $\left|p\right\rangle \otimes \left|k\right\rangle$ is the initial state which the electron has the momentum p and the photon has $\hbar k$, the Fock vector. $\left\langle p+\hbar k-\hbar k'\right| \otimes \left\langle \hbar k'\right|$ is the final state. Moreover, the red line in Fig.1 is the "scattered photon" which behaves as the radiation. It requires us to decide the final value of the momentum of the bremsstrahlung. This means it is necessary to integrate of the 4-momentum of the bremsstrahlung $\hbar k'_{\text{bremsstrahlung}}$.

$$\hbar k'_{\text{bremsstrahlung}} = \int d^4 k' \, \hbar k' \left| \mathcal{G}_{p,\hbar k}^{p+\hbar k-\hbar k',\hbar k'} \right|^2 \quad (5)$$

This method is difficult for the simulation of the laser-electron interaction.

In classical theory of Dirac [3], he suggested that the reaction force should be described by the retarded field A_{ret} and advanced field A_{adv}. These fields are satisfied with the equation; $\partial_{\mu}\partial^{\mu} A_{\text{full}}{}^{\nu} = \mu_0 j^{\nu}$.

$$A_{\text{full}} = \frac{1}{2}\left(A_{\text{ret}} + A_{\text{adv}}\right) \quad (6)$$

$$A_{\text{reaction}} = \frac{1}{2}\left(A_{\text{ret}} - A_{\text{adv}}\right) \quad (7)$$

Equation (4) means we can consider $A_{\text{ret}}/2$ is the photon field and $A_{\text{adv}}/2$ is the antiparticle field in QED method. The reaction force of the LAD theory is described by

$$f_{\text{reaction}}{}^{\mu} = -e\left(\partial^{\mu} A_{\text{reaction}}{}^{\nu} - \partial^{\nu} A_{\text{reaction}}{}^{\mu}\right) w_{\nu} . \quad (8)$$

This means the reaction force is defined by the potential A_{reaction} like an electromagnetic field. The potential in the Dirac equation is replaced as

$$A \mapsto A_{\text{laser}} + A_{\text{reaction}} \quad (9)$$

if the radiation reaction is considered without the electron-positron pair creation. Therefore, the Dirac equation with the radiation reaction is

$$\left\{ i\hbar\gamma^{\mu} \left[\partial_{\mu} - \frac{ie}{\hbar} \begin{pmatrix} A_{\text{laser}} \\ + \\ A_{\text{reaction}} \end{pmatrix}_{\mu} \right] - mc\mathbb{I} \right\} \psi = 0 . \quad (10)$$

We will show the detail of the radiation reaction in the classical and the quantum description in our presentation.

References

[1] K. Seto, H. Nagatomo and K. Mima, Plasma Fusion Res. **6**, 2404099 (2011).
[2] L. D. Landau and E. M. Lifshitz, *The Classical theory of fields* (Pergamon, New York, 1994).
[3] P. A. M. Dirac, Proc. Roy Soc. A **167**, 148 (1938).
[4] G.A.Mourou, C.P.J.Barry and M.D.Perry, Phys. Today **51**,22 (1998).
[5] R. P. Feynman, Phys. Rev. **76**, 749 (1949)
[6] R. P. Feynman, Phys. Rev. **76**, 769 (1949)