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Dependence of Nonlinear Evolution of Magnetosonic Waves on Ion Composition and Propagation Angle
非線形磁気音波のイオン組成と伝播角に対する依存性

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The dependence of nonlinear evolution of magnetosonic waves on ion composition and propagation angle θ is studied. First, the conditions necessary for KdV equation for the low-frequency mode in a two-ion-species plasma is analytically obtained. The upper limit of the amplitude of the low-frequency-mode pulse is expressed as a function of θ, density ratio, and cyclotron frequency ratio of two ion species. Next, with electromagnetic particle simulations, the nonlinear evolution of a long-wavelength low-frequency-mode disturbance is examined for various θs in two plasmas with different ion densities and cyclotron frequency ratios, and the theory for the low-frequency-mode pulse is confirmed. It is also shown that if the pulse amplitude exceeds the theoretical value of the upper limit of the amplitude, then shorter-wavelength low- and high-frequency-mode waves are generated.

1. Introduction

The presence of the multiple species ions influences the properties of magnetosonic waves. For example, in a two-ion-species plasma, the magnetosonic wave propagating perpendicular to a magnetic field is split into two modes; the low- and high-frequency modes. The frequencies of the low-frequency mode are in the range $0 < \omega < \omega_{-\mathrm{r}}$, where $\omega_{-\mathrm{r}}$ is the ion-ion hybrid resonance frequency [1],

$$\omega_{-\mathrm{r}} = [(\omega_{pa}^2\Omega_b^2 + \omega_{pb}^2\Omega_a^2)/(\omega_{pa}^2 + \omega_{pb}^2)]^{1/2}. \tag{1}$$

Here, the subscripts a and b indicate ion species, and Ω_j and ω_{pj} ($j = a$ or b) represent their cyclotron and plasma frequencies, respectively. The high-frequency mode has a finite cut-off frequency given by

$$\omega_{+0} = (\omega_{pa}^2/\Omega_a^2 + \omega_{pb}^2/\Omega_b^2)\Omega_a\Omega_b|\Omega_\mathrm{e}|/\omega_\mathrm{pe}^2. \tag{2}$$

Here, the subscript e refers to the electrons.

Although the dispersion curves of the high- and low-frequency modes are quite different in the long-wavelength region, Korteweg-de Vries (KdV) equations have been derived for both the low- and high-frequency modes [2]. The KdV equation for the high-frequency mode is valid for wave amplitudes $(m_e/m_i)^{1/2} \ll \varepsilon \ll 1$. The KdV equation for the low-frequency mode is valid when $\varepsilon < 2\Delta_\omega$, where Δ_ω is defined as

$$\Delta_\omega = (\omega_{+0} - \omega_{-\mathrm{r}})/\omega_{+0}, \tag{3}$$

The value of Δ_ω increases with increasing Ω_a/Ω_b, where $\Omega_a > \Omega_b$ is assumed. For a fixed Ω_a/Ω_b, Δ_ω has its maximum value when the ion charge densities are equal, $n_aq_a = n_bq_b$. In the nonlinear evolution of magnetosonic waves, Δ_ω is an important parameter[3]. In fact, electromagnetic particle simulations demonstrated that high-frequency-mode pulses are generated from a long-wavelength low-frequency-mode pulse if its amplitude exceeds $2\Delta_\omega$. In this study, we extend the above work to the case of oblique magnetosonic waves.

2. Theory for Low-frequency mode

We derive the condition for the KdV equation for the low-frequency mode to be valid, assuming that the waves propagate in the x direction in a magnetic field in a (x, z) plane, $\boldsymbol{B}_0 = B_0(\cos\theta, 0, \sin\theta)$. We consider the cases of $\theta < \theta_\mathrm{cl}$, where θ_cl is defined as $\cos^2\theta_\mathrm{cl} = 2\Delta_\omega/r$ with

$$r = \omega_{-\mathrm{r}}^2(\Omega_a^2 + \Omega_b^2)/(\Omega_a^2\Omega_b^2) - 1. \tag{4}$$

It is found that the linear dispersion relation of the low-frequency mode can be written as

$$\omega = v_\mathrm{A}k(1 + d_\mathrm{l}^2k^2/2). \tag{5}$$

for small wave numbers $k \ll k_\mathrm{inf}^{(\mathrm{l})}$, Here, $k_\mathrm{inf}^{(\mathrm{l})}$ and d_l are given by

$$\frac{k_\mathrm{inf}^{(\mathrm{l})}}{k_\mathrm{c}} = \left[\frac{3\sin^2\theta(r\cos^2\theta - 2\Delta_\omega)}{20\Delta_\omega - 2(r\cos^2\theta - 2\Delta_\omega)^2}\right]^{1/2}. \tag{6}$$

$$d_\mathrm{l} = |2\Delta_\omega - r\cos^2\theta|^{1/2}/(k_\mathrm{c}\sin\theta) \tag{7}$$

with $k_c = \omega_{-\mathrm{r}}/v_\mathrm{A}$.

As expected from Eq. (5), the nonlinear behavior of the low-frequency mode can be described by the KdV equation [4], and the characteristic wavenumber of the solitary pulse can be estimated as $k \sim \varepsilon^{1/2}/d_\mathrm{l}$. The dispersion form (5) is valid in the long wavelength region, $k \ll k_\mathrm{inf}^{(\mathrm{l})}$. Then, we obtain a condition for the amplitude of the rarefactive pulse as $\varepsilon \ll \varepsilon_\mathrm{max}^{(\mathrm{l}-)}$, where $\varepsilon_\mathrm{max}^{(\mathrm{l}-)}$ is defined as

$$\varepsilon_\mathrm{max}^{(\mathrm{l}-)} = \frac{3(r\cos^2\theta - 2\Delta_\omega)^2}{20\Delta_\omega - 2(r\cos^2\theta - 2\Delta_\omega)^2}. \tag{8}$$

The value of $\varepsilon_\mathrm{max}^{(\mathrm{l}-)}$ increases with decreasing θ from θ_cl. As Δ_ω increases, $\varepsilon_\mathrm{max}^{(\mathrm{l}-)}$ increases.

3. Simulation of Nonlinear Evolution

We study nonlinear evolution of oblique magnetosonic waves with numerical simulations, using a one-dimensional (one space coordinate and three velocity components), electromagnetic particle code with full ion and electron dynamics. We simulate H-T and H-He plasmas. The hydrogen-to-electron mass ratio is taken to be $m_{\rm H}/m_{\rm e} = 100$. Initially, we have a sinusoidal disturbance of the low-frequency mode with a wavelength L_x =4096Δ_g, where Δ_g is the grid spacing, propagating in the positive x direction. The amplitude of the magnetic field B_z is chosen to be $\delta B_z/B_0 = 0.1$.

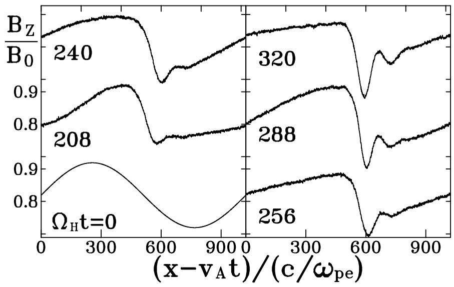

Fig. 1 Wave evolution of the low-frequency mode for $\theta = 55°$ in the H-T plasma with $n_{\rm H} = n_{\rm T}$.

We firstly study the rarefactive pulse of the low-frequency mode in the H-T plasma with $n_{\rm H}/n_{\rm T} = 1$, taking the propagation angle to be $\theta = 55°$. The value of $\varepsilon_{\rm max}^{\rm (l-)}$ for this case is 0.48. Figure 1 shows magnetic field profiles at various times. As a result of nonlinear evolution, some pulses are formed. These are the rarefactive pulses of the low-frequency mode. High-frequency-mode pulses are not found. At $\Omega_{\rm H}t = 320$, the left pulse has the amplitude $\varepsilon = (B_{\rm max} - B_{\rm min})/B_0 = 0.24$. Even though the maximum pulse amplitude ε is greater than $\delta B_z/B_0$ of the initial disturbance, it does not exceed $\varepsilon_{\rm max}^{\rm (l-)}$.

We next consider a case for which the KdV equation is not valid. Figure 2 shows magnetic field profiles at $\Omega_{\rm H}t = 208, 256$ and 320 and power spectrum for the case of $\theta = 60°$ for which $\varepsilon_{\rm max}^{\rm (l-)}$ =0.18. As the result of the wave steepening, rarefactive pulses are formed, and the amplitude of the left pulse exceeds $\varepsilon_{\rm max}^{\rm (l-)}$; for example, at $\Omega_{\rm H}t = 256$, the amplitude is $\varepsilon = (B_{\rm max} - B_{\rm min})/B_0 = 0.23$. After $\Omega_{\rm H}t = 256$, perturbations are generated behind the left pulse. The power spectrum $P(k, \omega)$ in Fig. 2 shows that the low-frequency-mode waves with the wavenumbers $k_{\rm inf}^{\rm (l)} < k \leq k_{\rm c}$ are created, in addition to the longer-wavelength waves with $k < k_{\rm inf}^{\rm (l)}$.

We finally simulate an H-He plasma with $n_{\rm He}/n_{\rm H} = 0.1$. Figure 3 shows the result of the case with $\theta = 66°$. The value of $\varepsilon_{\rm max}^{\rm (l-)}$ for this case is 0.005, which is much smaller than those for the cases of the above H-T plasmas. Power spectrum clearly shows that the high-frequency-mode waves with $\omega > \omega_{+0}$ and $k > k_{\rm c}$ are generated.

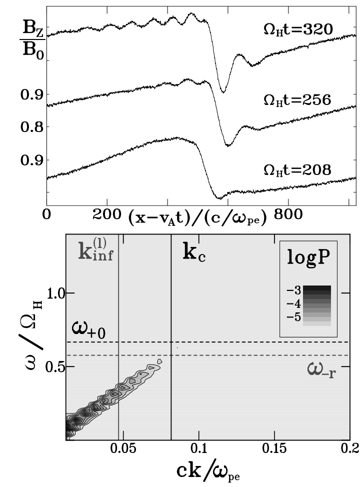

Fig. 2 Wave evolution and power spectrum of magnetic field for $\theta = 60°$ in the H-T plasma with $n_{\rm H} = n_{\rm T}$.

4. Summary

We have studied the nonlinear evolution of oblique magnetosonic waves. First, we analytically obtained the condition for the KdV equation for the low-frequency mode to be valid. The upper limit of the amplitudes $\varepsilon_{\rm max}^{\rm (l-)}$ has been given as a function of θ and Δ_ω. Next, electromagnetic particle simulations demonstrated that when the amplitude of the low-frequency-mode pulse exceeds $\varepsilon_{\rm max}^{\rm (l-)}$, shorter-wavelength low- and high-frequency-mode waves are generated.

References

[1] S. J. Buchsbaum, Phys. Fluids **3**, 418 (1960).
[2] M. Toida and Y. Ohsawa, J. Phys. Soc. Jpn. **63**, 573, (1994).
[3] M. Toida, H. Higashino, and Y. Ohsawa, J. Phys. Soc. Jpn. **76**, 104502 (2007).
[4] S. Irie and Y. Ohsawa, Phys. Plasmas **10** 1255 (2003).

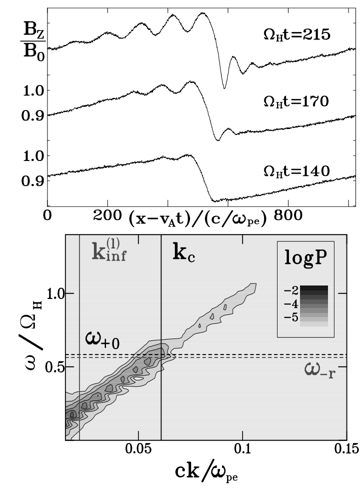

Fig. 3 Wave evolution and power spectrum of magnetic field for $\theta = 66°$ in the H-He plasma with $n_{\rm H} = 10n_{\rm He}$