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Beyond the Relativistic Magnetohydrodynamics: Numerical Scheme for Relativistic Resistive Radiation Hydrodynamics
相対論的抵抗性輻射磁気流体コードの数値解法

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We developed a numerical scheme for solving a set of Relativistic Resistive Radiation Magnetohydrodynamics (R3MHD) equations, which ensures a conservation of total energy and momentum of the matter, magnetic field and radiation. The electric resistivity is assumed to be istropic in the comoving frame. The radiation field is described by the 0th and 1st moment equations of the radiation transfer equation. The hyperbolic terms, whose timescale is characterized by the light crossing time in the relativistic plasma, are explicitly integrated using approximate Rieman solvers. Source terms, describing a magnetic diffusion and an exchange of energy and momentum between the plasma and radiation, are implicitly integrated using iteration method. This allows us to take a larger Cournat-Friedrichs-Lewy conditions when the magnetic Reynolds number is large or the plasma is optically thick. Our newly developed scheme would be applicable for a number of relativistic astrophysical phenomena.

1. Introduction

Relativistic flows are appeared in many astrophysical phenomena, such as the jets from microquasars and active galactic nuclei, pulsar winds, soft gamma-ray repeaters, core-collapse supernovae, and gamma-ray bursts. In many of these systems, the magnetic field has a crucial role in their dynamics. For example, the magnetic field connecting between the central star and the accretion disk, or, the different points of the accretion disks, are twisted and amplified due to the differential rotation, launching the jets [1-3]. The jets are also powered by the rotational energy of the blackhole via the Poynting flux [4]. The electric resistivity can play an important in these dynamics since it changes the topology of the magnetic fields. The finite electric resistivity can be the origin of the flares observed in the high-energy astrophysical phenomena [5-6].

The radiation field is also a key ingredient to determine the dynamics of relativistic phenomena. The radiation pressure force can facilitate the jet acceleration [7-9]. Recently the production of magnetically collimated, radiatively accelerated jet is modeled using non-relativistic radiation magnetohydrodynamic simulations [10]. Thus it is important to consistently take into account these effects to understand the relativistic phenomena.

In the Relativistic Resistive Radiation Magnetohydrodynamics (R3MHD), there are four timescales in the system: (i) the dynamical time sale, $t_{\rm dyn}$ (ii) light crossing time $t_{\rm c}$, (iii) damping timescale of electric fields $t_{\rm damp}$, and (iv) absorption/emission or scattering timescales, $t_{\rm ab}$ and $t_{\rm sc}$. The first two timescales are comparable in the relativistic fluids. The damping time scale $t_{\rm damp}$, characterizing the magnetic diffusion due to the ohmic dissipation, can be much shorter than the former two timescales when the magnetic Reynolds number is larger than unity. Also $t_{\rm ab}$ and $t_{\rm sc}$ can be much shorter than $t_{\rm dyn}$ and $t_{\rm c}$ in the optically thick medium. These facts indicate that R3MHD equations become stiff for the high magnetic Reynolds number and the large optical depth. Such stiff equations are difficult to explicitly integrate with time. In this paper, we developed an Explicit-Implicit scheme for solving R3MHD equations to overcome these problems. This is the first challenge for the R3MHD simulations in the world.

2. Outline of numerical scheme

In the following, we take the light speed as unity. The set of R3MHD equations is given by

$$\partial_\nu(\rho u^\nu) = 0, \ (1)$$
$$\partial_\nu T^{\mu\nu}_{\rm MHD} = G^\mu, \ (2)$$
$$\partial_\nu T^{\mu\nu}_{\rm rad} = -G^\mu, \ (3)$$
$$\partial_\nu F^{\mu\nu} = I^\nu, \ (4)$$
$$\partial_\nu \, {}^*F^{\mu\nu} = 0, \ (5)$$
$$I_\nu = \sigma F_{\nu\mu} u^\mu + q_0 u_\nu, \ (6)$$

where ρ, u^ν, $T_{\rm MHD}{}^{\mu\nu}$, $T_{\rm rad}{}^{\mu\nu}$, $F^{\mu\nu}$, I_ν, σ, q_0, are the proper mass density, bulk four velocity, energy momentum tensors of magnetofluids and radiation, Maxwell tensor of electromagnetic fields, four vector of the electric current, conductivity, and proper electric charge density, respectively. G^{ν} is the radiation four force. Equation (1) - (5) describes the time evolution of the systems, while equation (6) gives the Ohm's law. Another relation for the radiation field is needed to close the system, i.e., the closure relation. We utilize the general form of the closure relation as,

$$P_r^{ij} = D^{ij} E_r, (7)$$

where P_r^{ij} and E_r are the radiation stress and the radiation energy density, while D^{ij} is the so-called Eddington tensor, which is generally a function of radiation fields.

In our treatment, equation (1) - (5) is integrated using an operator-splitting method. Then, these equations are symbolically expressed by

$$\partial_t U + \nabla \cdot \boldsymbol{F} = 0, \ (8)$$
$$\partial_t U = S, \ (9)$$

where U, $\boldsymbol{F}$, S denote the conservative variable, flux and source term, respectively. The source term describes the magnetic diffusion and the gas-radiation interaction so that equation (9) can be stiff. We propose that equation (7) is explicitly integrated, while equation (9) is implicitly integrated using iteration method. The implicit scheme allows us to take the numerical time step δt larger than $t_{\rm damp}$, $t_{\rm ab}$ and $t_{\rm sc}$.

3. Results and Conclusions.

The first numerical test is the so-called shadow problem. The computation is performed in the 2-dimensional Cartesian coordinate. The gas distributes uniformly in space and the radiation field equilibrates with the gas. The optical depth of the system is about 0.01. There is a clump around the origin that the plasma density is 1000 times larger than that of the uniform matter. The corresponding optical depth is about 10. The radiation is injected at the left boundary (x=-5cm) with a constant luminosity. Figure 1 shows the colour contour of the radiation energy at the final state. In the upper panel, the Eddington approximation is assumed in the closure relation that the radiation field is isotropic in the momentum space. In the lower panel, the M-1 closure [10] is adopted, which allows the anisotropy of the radiation fields. We can see that the radiation energy density is uniform in space outside the clump with the Eddington approximation, while the shadow is successively reproduced behind the clump with the M-1 closure. We have to note that the absorption timescale in the clump is about 10 times shorter than the light crossing time. This means that the numerical solution can be unstable when the source term is explicitly integrated. In our numerical scheme, the implicit integration of source term allows us to take a large time step without numerical instability.

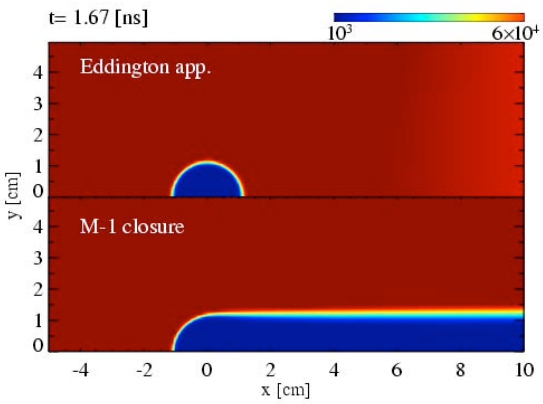

Fig.1. Numerical results of the shadow problems. Upper and lower panels correspond to results with the Eddington approximation and the M-1 closure. Colour shows the radiation energy density.

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