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A new self-similar solution for shock break-out of super-novae
超新星爆発Shock break-outに関する新自己相似解

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Although many researches are focused on the shock wave accompanying supernova explosion, the self-similar solution of the self-consistent explosion which taking into account both self-gravity and radiant heat conduction has never been addressed. A new self-similarity solution was found in terms of a novel with the help of the Lie-group method. This scheme is expected to be also applied to different problems with multiple heat conduction mechanism.

1. Introduction

Self-similar solutions play a crucial role in any branches of physics, in particular, for such fields as hydrodynamic phenomena in astrophysics. The self-similar solutions under different assumptions have indeed widened their applicability and enriched our qualitative understanding of physics.

A new self-similar solution describing spherical explosions of a gaseous sphere under both self-gravity and radiative diffusion is investigated in detail, where the diffusion is modeled by a power law with respect to density and temperature. The reduced dimensional eigenvalue problem is solved to show that there is a unique quantitative relation between the physical effects for self-similar dynamics. The resultant spatial and temporal behaviors are also determined uniquely, once the opacity is specified.

2. Basic equation and similarity ansatz

The one-dimensional spherical gas-dynamical equations with both self-gravity and diffusivity are

$$\frac{\partial \rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho u)=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial \rho}{\partial r}-\frac{\partial \varphi}{\partial r}, \tag{2}$$

$$\frac{1}{r^2}\frac{\partial \rho}{\partial t}\left(r^2\frac{\partial \varphi}{\partial r}\right)=4\pi G\rho, \tag{3}$$

$$\rho\left(\frac{\partial \epsilon}{\partial t}+u\frac{\partial \epsilon}{\partial r}\right)+\frac{p}{r^2}\frac{\partial}{\partial r}(r^2u) =\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\nu\frac{\partial T}{\partial r}\right), \tag{4}$$

where p is the pressure, ρ is the density, ϵ is the specific internal energy, u is the flow velocity, and φ is the gravitational potential. We assume an ideal gas equation of state in the form

$$\frac{(z+1)k_B}{\mu}T=\frac{p}{\rho}=(\gamma-1)\epsilon \tag{5}$$

where k_B is the Boltzmann constant, μ is the mean atomic mass, and γ is the specific heat ratio. Equation (4), described by the one-temperature model, includes the nonlinear heat diffusion term on the right-hand side, where we assume a power law dependence for the diffusion coefficient,

$$\nu=\nu_0T^n/\rho^m \tag{6}$$

with ν_0, m, and n being constants. For normal physical values, $n > 0$ and $m > 0$ are assumed.

To find a self-similar solution, we here introduce the following group transformation;

$$\begin{gathered} R(t)=At^{1/a}, \xi\equiv\frac{r}{R(t)}, \\ u=\frac{A}{a}t^{b/a}v(\xi), \\ T=\left(\frac{A}{a}\right)^2t^{c/a}\tau(\xi), \\ \rho=Bt^{-2}g(\xi), \\ \frac{\partial\varphi}{\partial r}=\frac{ABG}{a}t^{(e-1)/a}\Omega(\xi), \\ \Omega(\xi)\equiv 4\pi\int_0^{\xi}\xi^2g(\xi), \end{gathered} \tag{7}$$

where a, b, c, d, and e is the constant and $R(t)$ is the temporal characteristic scale length of the system; A and B are positive constants defining the scale of the radius and the density, respectively. These constants can be uniquely determined by substituting equation (1) - (4) such that the transformed system is the kept symmetric and thus has the same structure as the original one based on Lie's idea. Then, equation (1), (2), and (4) are reduced, respectively, to the following ordinary differential equations:

$$-(\xi - v)g' + (\mathrm{d} + \mathrm{v}^{\wedge\prime} + 2\mathrm{v} /\ \xi\,)g = 0 \qquad (8)$$

$$bv - (\xi - v)v' + {(g\tau)'}/{g} + {K_1\Omega}/{\xi^2} = 0 \qquad (9)$$

$$\frac{c\tau - (\xi - v)\tau'}{\gamma - 1} + \left(v' + {2v}/{\xi}\right)\tau = K_2 \frac{(\xi^2 g^{-m}\tau^n\tau')'}{g\xi^2} \qquad (10)$$

Since equation (3) is automatically satisfied, its reduced form does not appear in the above set of equations. As can be seen in equations (18) and (19), the present system is characterized by two positive dimensionless parameters, K_1 and K_2. Equations (8)-(9) are a second order ordinary equation system for g, τ, v, and the obvious boundary conditions at $\xi = 0$ are

$$\mathrm{v} = 0, \mathrm{g} = 0, \tau = 1\,, (\mathrm{g}\tau)' = 0 \qquad (11)$$

The last relation physically means that there is no pressure gradient at the center.

3. The self-similar solution

At a glance, the ODE system, together with boundary condition (11), seems to be closed mathematically. However one can easily find that numerical integration of the system produced a physically unacceptable picture under an arbitrary set of the values of K_1, K_2, and ξ_S , where ξ_S is the shock front parameter, such that the temperature suddenly drops to zero with the density being kept nonzero at a finite radius. Since the physically quantities are expected to change smoothly in space, it is conjectured that some special values of K_1, K_2, and ξ_S which are still unknown, can give such a physically acceptable picture. Therefore the present system is supposed to be a two-dimensional eigenvalue problems treated in previous work.

To determine a unique set of parameter, we need two more physical conditions. The first one is that the temperature should converge to zero simultaneously with increasing radius. Note that self-similar models based on the isothermal or adiabatic assumptions do not need energy equation (4), and the system is described in terms of only a single unknown constant, ξ_S, which is determined by this first condition. The second one is that the velocity should converge to zero at $\xi \to \infty$.

Although we can determine the value of ξ_S, and K_2 by using both condition, K_1 is not still unknown. Figure 1 shows the eigenstructure of the self-similar solution for the reference case, m=1, n=13/2, γ =1.3, and K_1=0.01.

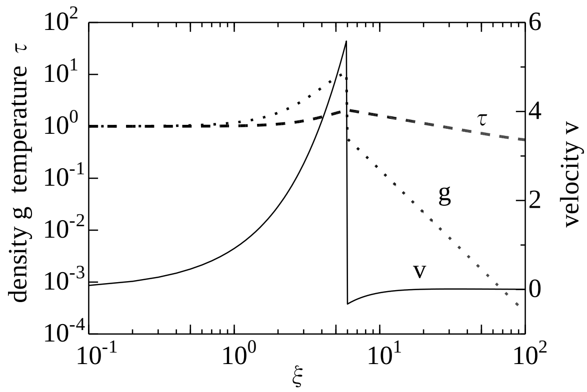

Fig.1. the self-similar solution for reference case, m=1, n=13/2, γ =1.3, and K_1=0.01

4. Conclusion

Under the appropriate similarity ansatz and variable transformations, the hydrodynamic system is reduced to the novel two-dimensional eigenvalue problem. The physical implication is that a unique quantitative relation between the gravity and the heat diffusivity indwells in the self-similar dynamics.

References

[1] Landau, L. D., & Lifshitz, E.M 1959, Fluid Mechanics(London: Pergament)

[2] Sedov, L. I. 1959, Similarity and Dimensional Methods in mechanics(New York: Academic)

[3] Zeldvich, Ya. B., & Raizer, Yu. P. 1960, Physics of Shock Waves and High Temperature Hydrodynamic Phenomena(New York: Academic)

[4] G. Guderley, Luftfahrforschung 19, 302 (1942)

[5] M. Murakami, K. Nishihara, and T. Hanawa, Astrophys. J. 607, 879 (2004)