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Multi-Moment Advection Scheme for Vlasov Simulations
ブラソフシミュレーションのためのマルチモーメント移流法

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We present a new numerical scheme for solving the advection equation and its application to Vlasov simulations. The scheme treats not only point values of a profile but also its zeroth to second order piecewise moments as dependent variables, and advances them on the basis of their governing equations. We show that the scheme provides quite accurate solutions within reasonable usage of computational resources compared to other existing schemes. Applications of the scheme to Vlasov simulations are presented with some benchmark tests.

1 Introduction

The Vlasov simulation, which directly discretizes the Vlasov equation on grid point in phase space, has been proposed for a collisionless plasma simulation method, to remedy problems inherent to the common Particle-In-Cell (PIC) simulation. A number of numerical schemes of the advection equation have been proposed for the Vlasov simulation and succeeded in applying to the electrostatic Vlasov-Poisson simulations thus far (e.g., [1]). However, the application to the electromagnetic Vlasov simulation of magnetized plasma is still limited, mainly owing to the difficulty in solving the gyro motion around the magnetic field. In this paper, we propose a new numerical scheme for the advection equation, specifically designed to solve the Vlasov equation in magnetized plasma.

2 Multi-Moment Advection Scheme

The present scheme considers the advection of a profile $f(\boldsymbol{x},t)$ and its zeroth to second order moments defined as

$$\boldsymbol{M}^m = \frac{1}{m!}\int \boldsymbol{x}^m f d\boldsymbol{x}, \quad (m=0,1,2)\,. \tag{1}$$

In one dimension, their governing equations are written as

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x}(uf) = 0, \tag{2}$$

$$\frac{\partial M^0}{\partial t} + \int dx \frac{\partial}{\partial x}(uf) = 0, \tag{3}$$

$$\frac{\partial M^m}{\partial t} + \frac{1}{m!}\int dx \frac{\partial}{\partial x}(ux^m f) = \frac{1}{(m-1)!}\int ux^{m-1} f dx, \quad (m=1,2)\,, \tag{4}$$

where u is the velocity. To solve a set of these equations, the scheme treats the point value of the profile f_i and the piecewise moments as dependent variables,

$$M^m_{i+1/2} = \frac{1}{m!}\int_{x_i}^{x_{i+1}} x^m f dx, \quad (m=0,1,2)\,, \tag{5}$$

and constructs a piecewise interpolation for f in a cell with a polynomial, $F_i(x)=\sum_{k=1}^{5} kC_{k;i}(x-x_i)^{k-1}$. The coefficients $C_{k;i}$ are explicitly determined from the dependent variables at the upwind position as constraint. Then the variables are advanced on the basis of their governing equations (2)-(4) with the semi-Lagrangian method. The multi-dimensional scheme is designed in a similar way. The scheme is termed as the "Multi-Moment Advection (MMA)" scheme [2].

3 Benchmark Tests

Figure 1 shows the two-dimensional solid body rotation and advection problem of a symmetric gaussian profile, solved by the MMA, CIP-CSL2 [3], and backsubstitution [4] schemes. While other schemes

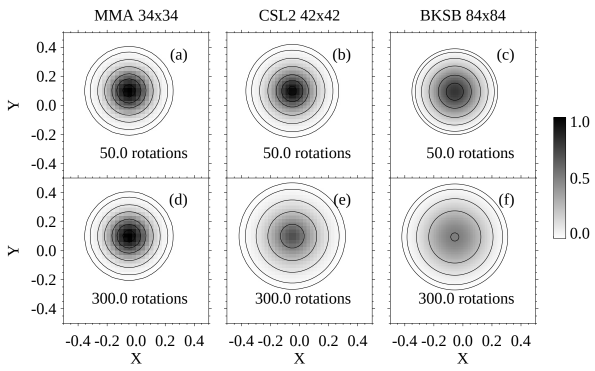

Figure 1: Two-dimensional solid body rotation and advection of a symmetric gaussian profile after (a,b,c) 50 and (d,e,f) 300 rotations, calculated with (a,d) the MMA (34×34 grid points), (b,e) the CIP-CSL2 (42×42), and (c,f) the backsubstitution (84×84) schemes.

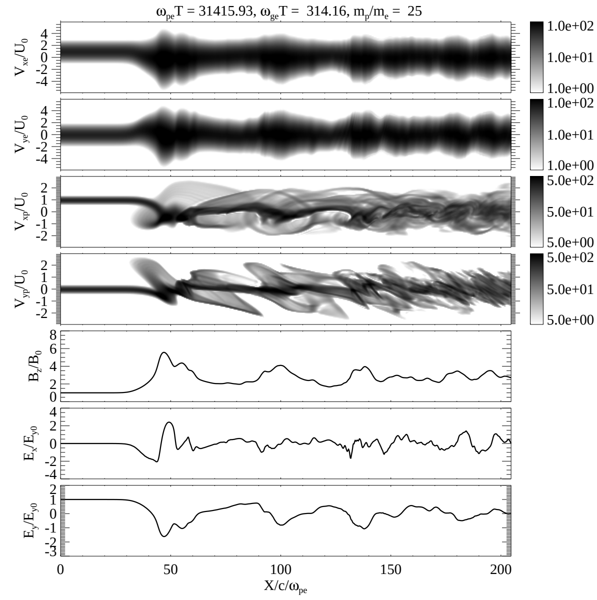

Figure 2: One-dimensional electromagnetic Vlasov simulation of perpendicular shock waves. (a,b) The electron phase space distributions in (x, v_x) and (x, v_y). (c,d) The proton phase space distributions in (x, v_x) and (x, v_y). (e,f,g) The electromagnetic field (B_z, E_x, E_y).

show serious numerical diffusion after several tens of rotation periods, the MMA scheme completely preserves the profile even after hundreds of rotation periods. This is a very important property for Vlasov simulations of magnetized plasma. Note that since the numbers of dependent variables are different among the three schemes, we use the different number of grid points so that the total memory usage is equal.

Applying the MMA scheme to the electromagnetic Vlasov simulation, we simulate the one-dimensional, strictly perpendicular collisionless shock waves. The simulation parameters are as follows; a proton to electron mass ratio $m_p/m_e = 25$, a ratio of the electron plasma to gyro frequency $\omega_{pe}/\omega_{ge} = 100$, electron and proton plasma beta values $\beta_e = \beta_p = 1.0$, and the spatial grid size $\Delta x = 20\lambda_D$, where λ_D is the Debye length. Figure 2 shows the plasma phase space distribution and the electromagnetic fields. An Alfvén Mach number of the resulting shock wave is ~ 7.5 measured in the shock rest frame. The simulation describes fundamental properties of the perpendicular collisionless shock such as the electrostatic shock potential and the so-called reflected ions, and satisfies the Rankine-Hugoniot conservation laws.

4 Summary

We have presented a new numerical scheme for solving the advection equation and the Vlasov equation. The scheme has a quite high capability necessary for the Vlasov simulation of magnetized plasma. The application of the scheme to the electromagnetic Vlasov simulation of collisionless shock waves has been presented. Although the grid size is much larger than the Debye length, the simulation is stable and the fundamental shock properties are well described. This advantage over an explicit PIC simulation enables us to perform large-scale plasma kinetic simulations.

Acknowledgement

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