Sheath stability conditions for PIC simulation
PICシミュレーションにおけるシーク安定条件

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Particle in Cell (PIC) simulation is a very powerful tool to study the sheath structure. But careless choice of geometry parameter easily lead to unstable and unrealistic sheath behavior. In this work, 1D simulation results are compared with previous reported kinetic model results. In order to apply PIC simulation to more complicated probe system, 2D simulation and its stability are also studied. From these study, importance of careful simulation model for simultaneous simulation of sheath fine structure and large scale bulk plasma is confirmed.

1. Introduction
In order to study the sheath, various boundary condition of not only the solid surface side but also the plasma upflow side must be considered. In analytical model, moreover, calculation geometry must be divided into quasi-neutral presheath and collisionless sheath, whose solutions must be connected at a singular point (so-called sheath boundary) in a rather artificial manner. Moreover analytical model often assumes that electrons obey Maxwell-Boltzmann relation and that ion’s motion can be described by so-called free-fall model, which is not realistic in many plasma fields.

Particle simulation is a very powerful tool to study the sheath problem, since it is free from the above mathematical procedure. But careless choice of geometry parameter easily lead to unstable and unrealistic sheath behavior. In the present study, Particle in Cell (PIC) simulation by using Berkeley code (XOOPIC) is done [1]. Firstly, 1D simulation results are compared with the kinetic model results. In order to apply PIC simulation to more complicated probe system [2], 2D simulation and its stability are also studied.

2. Kinetic theory of the sheath
Here we restrict the situation to stationary 1-dimensional case for simplicity [3]. Plasma particles is injected from the plasma boundary and, if they reach the right wall boundary, they are absorbed perfectly. Potential at plasma boundary is set to be zero and negatively biased externally at wall boundary.

Fig.1: Condition for sheath establishment. Horizontal axis is the Absolute value of electric field (|E_s|) at source boundary. Vertical axis is the length where the potential reaches assigned solid boundary value.

We also neglect collision and particle sources (sinks) in the simulation geometry. Distribution function of particle spieces j obeys 1D steady collisionless Boltzmann equation.

\[ v_x \frac{\partial f_j(x, v_x)}{\partial x} + \frac{-q_j}{m_j} \frac{d\Phi}{dx} \frac{\partial f_j(x, v_x)}{\partial v_x} = 0 \tag{1} \]

By integrating the solution of eq (1) with \( v_x \), density is obtained as the function of normalized potential \( \phi = e(\Phi_s - \Phi)/T_e \) and \( \tau = T_i/T_e \). Potential \( \phi \) obeys normalized Poisson equation:

\[ -\frac{d\phi}{ds} = E(s), \quad \frac{dE}{ds} = \rho(s) = (n_i - n_e)/n_{es}, \tag{2} \]

where \( s = x/\lambda_D \).
Although eq. (2) is very stiff, solutions with different boundary electric field strength $E_s = \left(\frac{d\phi}{ds}\right)$ can be obtained. Potential profile obtained is found to be strongly dependent on precise value of boundary electric field. When $E_s = -1.30$, potential reaches $\phi_w$ at $s \sim 10$. But when $|E_s|$ becomes smaller by only 3%, flat region of potential grows and simulation geometry size becomes larger by factor of three. As shown in Fig. 1, such $|E_s|$ corresponds to the limit of stable sheath formation and, for smaller $|E_s|$ value, no stable sheath potential can not be obtained.

3. Simulation result
Firstly, a very simple simulation geometry is examined in Cartesian (x–y) Coordinate to compare with the kinetic model results. The simulation domain is a rectangle, and homogeneity along y-direction is kept. Left boundary at $x = 0$ is the plasma source and its potential is assumed to be zero. Right boundary at $x = L_x$ is the conductor, whose potential is kept to be -100[V].

If the domain size ($L_x$) is not so large, the XOOPIC simulation reaches the steady state and S–shape potential profile is obtained. When $L_x$ increases, however, the situation changes. Potential profile shows small fluctuations and its amplitude becomes large with $L_x$. Eventually, for $L_x/\lambda_D > 30$, potential profile oscillates and simulation does not reach the steady state solution. In this simulation, plasma density is determined by the input value of source boundary current. Even keeping $L_x$ constant, by increasing this current, Debye length $\lambda_D$ becomes small and sheath structure becomes unstable. Fig. 2 shows this tendency.

Recently, very complicated probe heads are proposed to extend the parameters to be measured. The sheath structure around these probe head also becomes complicated. Figure 3 shows the example of 2D-potential profile around the rim region of cylindrical probe head, which is set at right-bottom of this figure. Although the distance between probe and boundary in the z-direction is small than the critical value obtained with 1D simulation, the distance in the r-direction is somehow large. So large potential oscillations are observed and the sheath is unstable. In order to make optimize the probe structure[2], the gap in r-direction must be reduced than this model geometry.

Fig.2: Example of 1D sheath simulation for different plasma density. When source boundary current is large, plasma density is also large.

Fig.3: An example of 2D unstable sheath. Very large potential oscillation is observed at left-bottom of the figure.

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References