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Analysis of classical impurity transport in fusion plasmas by Monte-carlo Binary Collision Model
モンテカルロ 2 体衝突モデルによる核融合プラズマ中での不純物輸送の検討

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A numerical model for classical/neo-classical cross field diffusion of impurity ions in the magnetic fusion devices are being developed based on the Binary Collision Monte-Carlo Model. As a first step, the model validation has been done for a simple case with a uniform magnetic field. Reasonable agreement between the numerical result and the theoretical has been obtained. In addition to the conventional self-diffusion, the cross field transport towards the direction of the background density gradient predicted by the theory is also observed in the simulation.

1. Introduction

Understanding and control of impurity transport is one of the important issues to reduce the impurity radiation in the fusion core plasma region. Especially, impurity accumulation due to the neo-classical transport in the magnetic confinement devices, such as tokamaks, i.e., cross field transport by Coulomb collisions of impurity ions with background fuel ions (hydrogen) is one of possible causes. Impurity core penetration of tungsten impurity ions has been reported in the experiment during ICRF heating [1]. The reason, however, has not been clearly understood yet up to now.

The purpose of the present study is to examine whether the neo-classical transport process of impurity ions is correctly incorporated in the impurity transport code -"IMPGYRO" [2]. As a first step, we have validated impurity cross field diffusion in a uniform magnetic field in the IMPGYRO-code by comparing with a classical theory of cross field diffusion [3].

2. Cross-Field Diffusion of Charged Particles [3]

We first briefly summarize the classical theory of cross field diffusion based on Ref. 3. Consider now two types of particle, which we denote by subscript 1 and 2. The general expression for the flux $F_1(X)$ of guiding centers of type 1 due to stochastic process, i.e., Coulomb collisions of particles 1 and 2, is given by,

$$F_1 = N_1(X)\frac{<\Delta X_1>}{\Delta t} - \frac{1}{2}\frac{\partial}{\partial X}\frac{[N_1(X)<(\Delta X_1)^2>]}{\Delta t}, \qquad (1)$$

where $N_1(X)$ and ΔX_1 are the density of the guiding center of the particle species 1, and the step of the guiding center.

The step ΔX_1 is related to the change in the *y*-component of the velocity as

$$\Delta X_1 = \Delta v_{1y} / \omega_{1c}. \qquad (2)$$

Under the uniform magnetic field B_z and the assumption that the both particle species are magnetized with their Larmor radii $r_{L1} = v_1/\omega_{c1}, r_{L2} = v_2/\omega_{c2}$. Here, v_i and $\omega_{ic} = e_i B_z / m_i$ (e_i :electric charge, m_i :mass) are the particle speed and the cyclotron frequency of the *i*-th species, respectively. For a guiding center of particle 1 at X , the associated particle 1 is at $x = X - v_{1y}/\omega_{1c}$. The density of particles 2 at x is the same as density of guiding centers of particles 2 at $x + v_{2y}/\omega_{2c}$ $= X - v_{1y}/\omega_{1c} + v_{2y}/\omega_{2c}$. Hence, the probability per unit time that a particle 1 with guiding center at X will be involved in a collision, with scattering into solid angle $d\Omega$ is given by

$$\begin{aligned} &P(v_1, v_2, \Omega) d^3v_1 d^3v_2 d\Omega = \\ &d^3v_1 d^3v_2 g_1(v_1) g_2(v) N_2(X+\delta) v\sigma(\Omega) d\Omega, \end{aligned} \qquad (3)$$

where g_1 , g_2 $\sigma(\Omega)$, and v are the normalized velocity distributions, the differential scattering cross section of Coulomb collision, and the relative velocity $v = |v_1 - v_2|$, respectively. In addition, δ is given by $\delta = v_{2y}/\omega_{2c} - v_{1y}/\omega_{1c}$ from Eq. 2.

Using Eq. 3, we can calculate the first and the second moment of ΔX_1 in Eq. 1 as follows:

$$<\Delta X_1> = \alpha v_{12} r_{L1}^2 (1 + \frac{e_1}{e_2}) \frac{1}{N_2} \frac{\partial N_2}{\partial X} \Delta t, \qquad (4)$$

$$<(\Delta X_1)^2> = 2\alpha v_{12} r_{L1}^2 \Delta t. \qquad (5)$$

Here, α is a numerical factor depending on the mass m_1 and m_2 , and v_{12} is the collision frequency. It should be noted that Eq. 5 gives the self-diffusion of the particle species 1, while Eq. 4 gives the transport of the particle species 1 towards the density gradient of the species 2.

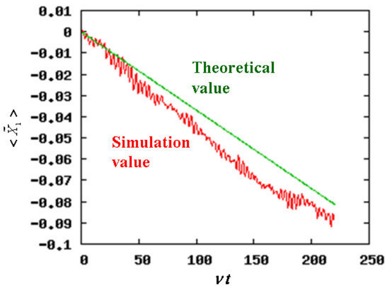

Fig. 1 Time evolution of the average position $<X_1>$

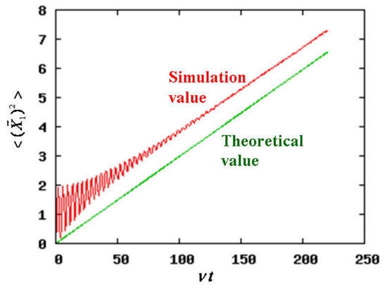

Fig. 2 Time evolution of the variance $<(X_1)^2>$

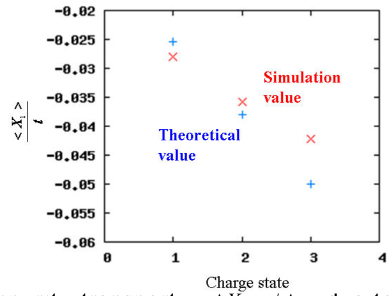

Fig. 3 Impurity transport $<\Delta X_1>/\Delta t$ due to background density gradient as a function of impurity charge state.

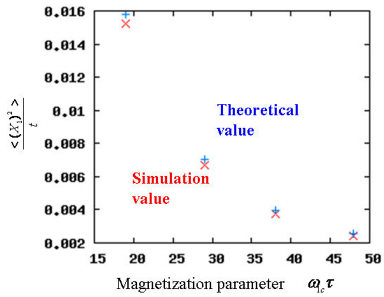

Fig. 4 Self-diffusion coefficients $<(\Delta X_1)^2>/\Delta t$ in comparison with the theoretical value

3. Numerical Model

The trajectory of each test particle is followed by Boris-Buneman algorithm [4]. In order to calculate the velocity change $\Delta \boldsymbol{v}$ due to Coulomb collision, the Binary Collision Monte-Carlo (BCM) model has been used, i.e., the scattering angle Θ and Φ are given in the following manner; $\Phi = 2\pi U$, $\Theta = 2\arctan\delta$, where U is a uniform random number and δ is sampled from the normal distribution with the mean value $<\delta>$ and the variance $<\delta^2>$ [5],

$$<\delta>=0,\quad <\delta^2>=\frac{N_2(e_1e_2)^2\ln\Lambda}{8\pi\varepsilon_0^2M^2v^3}\Delta t_c. \tag{6}$$

Here $\ln\Lambda$ is Coulomb logarithm, M is the reduced mass of the species 1 and 2, ε_0 is the permittivity of vacuum and Δt_c is the time-duration between two successive binary collision. The particle density N_2 in Eq. 6 is evaluated at the actual position of each test particle.

4. Results and Discussion

Test simulations have been done under following conditions as a reference case: 1) particle species 1: tungsten (W) with charge state Z=1 as particle species 1, 2) particle species 2: background Deuterium (D), with the density scale length $N_2/(dN_2/dx)=-0.1$. 3) The initial velocity of the species 1and the velocity of background species 2 are sampled from the Maxwell velocity distribution with a same temperature, and 4) uniform magnetic field ($B_z=5\mathrm{T}$).

Figure 1 and 2 show the simulation results of $<\tilde{X}_1>$ and $<(\tilde{X}_1)^2>$. The results are obtained by taking an ensemble average over all the test particles (10^5 particles) and normalized by Larmor radius.

The theoretical values $<\tilde{X}_1>/\nu t$ in Fig. 1 and $<(\tilde{X}_1)^2>/\nu t$ in Fig. 2 are obtained from Eq. 4 and from Eq. 5, respectively.

Figure 3 shows the simulation results $<\Delta X_1>/\Delta t$ for the different charge state Z of the species 1. Figure 4 compares the simulation results $<(\Delta X_1)^2>/\Delta t$ with the theoretical values for the different values of the magnetic field ($B=2\mathrm{T}\sim5\mathrm{T}$). The results are plotted as a function of the magnetized parameter $\omega_{1c}\tau$ $(=\omega_{1c}/\nu_{12})$. The numerical values of $<\Delta X_1>/\Delta t$ and $<(\Delta X_1)^2>/\Delta t$ in Fig. 3 and 4 are obtained by a linear fitting of the time evolution of $<X_1>$ and $<(X_1)^2>$.

5. Conclusion and future study

As seen from Figs. 1-4, reasonable agreement has been obtained between the numerical and theoretical results. The model improved in the present study will be incorporated in the IMPGYRO code. Neo-classical cross-field diffusions in realistic tokamak magnetic configuration will be studied in the future.

Reference

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