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Comparison of turbulent transport models in tokamak plasmas
トカマクプラズマにおける乱流輸送モデルの比較

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In tokamak plasmas, the turbulent transport that causes anomalous transport is a non-linear phenomena. The strong non-linearity of turbulent transport models sometimes causes numerical instability in transport simulation as a stiff problem. In this case, we have to take a very small time step for stable calculation and the simulation requires very long computation time. We have introduced a numerical scheme proposed by Pereverzev and show that the convergence is strongly improved for a simple diffusion equation with a non-linear diffusion equation. This scheme is implemented into the transport simulation code TASK/TR and simulation results for several transport models are compared with experimental observations.

1 Introduction

In tokamak plasmas the transport phenomena is classified into two categories, neoclassical transport and anomalous transport. Neoclassical transport is caused by coulomb collisions and magnetic mirror due to the tokamak torus configuration. Many experiments have shown, however, that the transport exceeds the amount which is predicted by the neoclassical theory. This increased transport is called anomalous transport and dominant in actual tokamak devices. Anomalous transport is considered to be caused by turbulent transport, which can be driven by the microscopic elect-magnetic fluctuations coming from pressure and current density gradients. Turbulence transport have not been figured out fully, and a number of models describe it have been proposed.

The turbulent transport that causes anomalous transport is a non-linear phenomena. The strong non-linearity of turbulent transport models sometimes causes numerical instability in transport simulation as a stiff problem. In this case, we have to take a very small time step for stable calculation and the simulation requires very long computation time. Pereverzev has proposed [1] a numerical scheme strongly improves the numerical convergence.

In our simulation of tokamak plasmas we use TR module of integrated transport analysis code TASK [2]. The TASK/TR module assumes 1-D diffusive transport model on the ground of flux surface average. The purpose of this paper is to introduce the improved numerical scheme into the TASK/TR module and compare several transport models including stiff ones with experimentally observed profiles.

2 Basic Equation

The basic equations of TASK/TR are following three diffusive equations:

$$\frac{\partial}{\partial t}(n_s)V' = -\frac{\partial}{\partial \rho}(V'\Gamma_s) + S_s V', \tag{1}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}n_s T_s V'^{5/3}\right) = -V'^{2/3}\frac{\partial}{\partial \rho}(V' Q_s) + S_{\mathrm{E}s}V'^{5/3}, \tag{2}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial \rho}\right) = \frac{\partial}{\partial \rho}\left[\frac{\eta_\parallel}{\mu_0}\frac{I}{V'\langle R^{-2}\rangle}\frac{\partial}{\partial \rho}\left(\left\langle\frac{|\nabla\rho|^2}{R^2}\right\rangle\frac{\partial \psi}{\partial \rho}\right) - \frac{\eta_\parallel}{I\langle R^{-2}\rangle}\langle (J_{\mathrm{CD}}+J_{\mathrm{BS}})B_\phi\rangle\right], \tag{3}$$

$$\Gamma_s = \langle|\nabla\rho|\rangle n_s V_s - \langle|\nabla\rho|^2\rangle D_s\frac{\partial n_s}{\partial \rho}, \tag{4}$$

$$Q_s = V_{\mathrm{E}s} n_s T_s - \langle|\nabla\rho|\rangle\chi_s\frac{\partial(n_s T_s)}{\partial\rho} + \left[-\langle|\nabla\rho|\rangle\left(\frac{5}{2}D_s - \chi_s\right)T_s\frac{\partial n_s}{\partial\rho}\right], \tag{5}$$

where s denotes the particle species, n_s is the particle density, T_s the temperature , Γ_s the particle flux, Q_s the heat flux, D_s the particle diffusivity, χ_s the thermal diffusivity, $V_{\mathrm{E}s}$ the sum of the particle pinch velocity V_s and the heat pinch velocity, ψ the poloidal magnetic flux, $\eta_\parallel$ the parallel neoclassical resistivity, μ_0 the permeability of free space, $I = B_\phi R$ where B_ϕ the toroidal magnetic field, R is the major radius, j_{CD} and J_{BS} are the induced current density and the bootstrap current density, V' the derivatives of the volume enclosed by a flux surface with respect to the normalized minor radius ρ and $\langle|\nabla\rho|\rangle$ and $\langle|\nabla\rho|^2\rangle$ are geometric quantities.

3 Stable numerical scheme

Some turbulent transport model such as GLF23 [3] are numerically stiff and when using these models in transport simulations we have to decrease the size of time step considerably, which is very time-consuming. We, however, can avoid this problem if the virtual diffusive term is added to transport equation.

We explain this concept using a simple diffusion equation in cylindrical coordinates

$$\frac{\partial u}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho D \frac{\partial u}{\partial \rho}\right) + S. \qquad (6)$$

Fig.1 shows the gradient-flux relation of a typical stiff transport model. The solid line shows that the diffusive coefficient and the flux increase abruptly when the amplitude of the gradient exceeds the threshold $\eta_{\rm cr}$. Transport models evaluate the diffusion coefficients as $D_{\rm eff}$ illustrated by the dotted line in Fig.1. Provided that the gradient is η_0 at a certain time step, the transport model returns the value $D_{\rm eff}(\eta_0)$. If the size of time step in calculation is not sufficiently small, then the transport code evaluates the gradient as $\eta_0^{'}$, and next the transport model returns much higher $D_{\rm eff}$ than previous value. This leads to numerical instability and the calculation doesn't come to converge numerically.

Now we consider the modified equation

$$\frac{\partial u}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \tilde{D} \frac{\partial u}{\partial \rho} - \rho \bar{V} u\right) + S, \qquad (7)$$

where $\tilde{D} = D_{\rm eff} + \bar{D}$, $\bar{V} = \bar{D}(\partial u/\partial \rho)/u$, and $\bar{D}$ is the arbitrary virtual diffusive coefficient. Eq.(7) is identical to Eq.(6) mathematically, but the non-vanishing term remains if $\tilde{D}$ is calculated at $t+\tau$ implicitly while $\bar{V}$ is calculated at t, where t is certain time step and τ is the size of time step. The gradient-flux relation of Eq.(7) is shown by the dash line in Fig.1, and obviously the instability will not be caused and the calculation will converge straightforwardly.

4 Numerical Results

We calculated Eq.(6) and Eq.(7) as preliminary calculation by means of finite difference method. The transport model is assumed to be stiff as is shown Fig.1, and we set $D_0 = 0.01$, $D_1 = 0.1$, $S = 1.0$, $\bar{D} = 0.05$. Then we found that the iteration using Eq.(6) does not converge at the size of time step larger than $\Delta t = 10^{-2}$, while that using Eq.(7) does converge even when the size of time step is $\Delta t = 10^{0}$. Fig.2 shows the comparison of the dependence of the number of iterations on the size of time step, from which we found that this scheme is validated in the preliminary calculation.

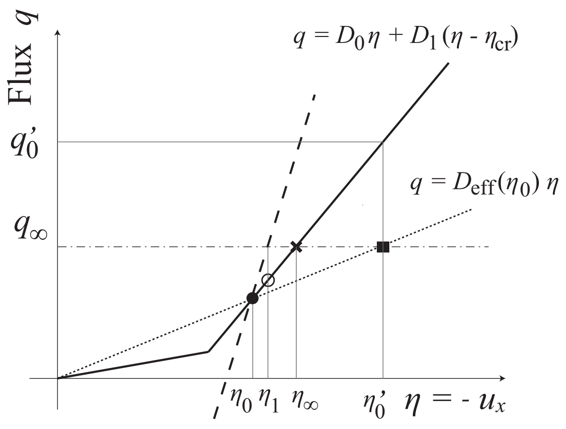

Figure 1: The gradient-flux relation of a typical stiff transport model. The dash line shows the relation of improved equation.

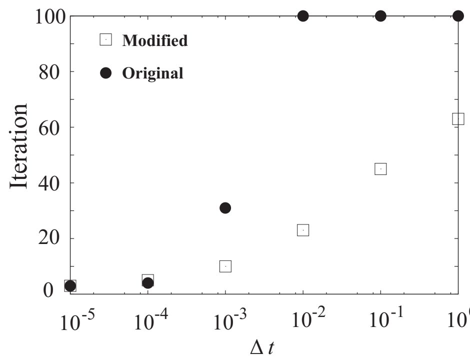

Figure 2: Comparison of the number of iteration.

5 Comparison of Transport Models

We will present the simulation result of actual tokamak plasmas with several turbulent transport models using improved TASK/TR.

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