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MHD equilibrium including a static magnetic island for the reduced MHD equations in straight heliotron plasmas

直線ヘリオトロン配位における静的磁気島を含んだ簡約化MHD方程式に対するMHD平衡

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MHD equilibria including static magnetic islands for the reduced MHD equations in straight heliotron plasmas are studied. The equilibria are obtained by solving the simultaneous equations of the constant pressure along each field line and of the force balance. There exist two kinds of solutions depending on the numerical method in solving the equation of the constant pressure along each field line. One is the pressure profile that is locally flat at not only the O-point but also the X-point, which is obtained by employing a diffusion equation parallel to the field line. The other is the profile that is locally flat only at the O-point, which is obtained by employing a field line tracing method.

1. Introduction

We study the interaction between static magnetic islands generated by an external field and resistive interchange modes by using the reduced MHD (magnetohydrodynamics) equations[1] in straight heliotron plasmas. In the previous work, a profile corresponding to nested flux surfaces was employed for the equilibrium pressure. In this case, interchange modes grow despite the existing static islands. The island width is changed by the nonlinear saturation of the interchange mode[2,3].

This pressure profile, however, does not satisfy the equilibrium equations with the magnetic field including the islands. The equilibrium pressure consistent with the topology of the static islands should be deformed to have local flat structure at the island region. The local flat structure is expected to affect the growth of the interchange mode. For the study of the deformation effect on the stability, equilibria with the pressure profile consistent with the static islands are needed. In the present study, we calculate the equilibria by solving the simultaneous equations of the constant pressure along each field line and of the force balance.

2. Simultaneous equations for equilibrium

MHD equilibria including a static magnetic island with the mode number of (m,n)=(1,1) are studied by using the reduced MHD equations in a straight heliotron configuration. Here, m and n are the polidal and toroidal mode numbers, respectively. These equations are suitable for the analysis of such low mode number physics. The reduced MHD equations are composed of Ohm’s law, vorticity equation and the pressure equation for the ploidal flux $\Psi(r,\theta)$, the stream function $\Phi(r,\theta)$ and the plasma pressure $P(r,\theta)$. The normalized equations in the cylindrical coordinates are given by

$$\frac{\partial \Psi}{\partial t} = -\boldsymbol{B}\cdot\nabla\Phi + \frac{1}{S}J_z, \qquad (1)$$

$$\frac{d\nabla_\perp^2\Phi}{dt} = -\mathbf{B}\cdot\nabla J_z + \frac{1}{2\epsilon^2}\nabla\Omega\times\nabla P\cdot\hat{z} \qquad (2)$$

and

$$\frac{\partial P}{\partial t} = (\hat{z}\times\nabla\Phi)\cdot\nabla P. \qquad (3)$$

Here, the magnetic field is expressed as $\mathbf{B} = \hat{z} + \hat{z}\times\nabla\Psi$. The equilibrium corresponding to Eq.(1)-(3) with static islands has to satisfy the following two equations. One is the force balance equation given by

$$-\mathbf{B}\cdot\nabla J_z + \frac{1}{2\epsilon^2}\nabla\Omega\times\nabla P\cdot\hat{z} = 0, \quad (4)$$

which is obtained by $\partial/\partial t = \Phi = 0$ in Eq.(2). The other is the constraint that the pressure is constant along field line with arbitrary topology,

$$\boldsymbol{B}\cdot\nabla P = 0. \quad (5)$$

Equations (4) and (5) are the simultaneous equations for Ψ(r, θ) and P(r, θ).

3. Calculation methods

Equations (4) and (5) are separately solved as in the case of the HINT code[4]. In the first step, P satisfying Eq.(5) is solved with Ψ fixed. In the second step, Ψ satisfying Eq.(4) is solved with P fixed. In the first step, we employ two methods. One is the method utilizing the diffusion equation parallel to the field line

$$\frac{\partial P}{\partial t} = \kappa_{\parallel}(\boldsymbol{B}\cdot\nabla)(\boldsymbol{B}\cdot\nabla)P. \quad (6)$$

We follow the time evolution of P from the initial profile $P_{sym}(r)$. The solution is obtained when the steady state is achieved. The other is the method tracing the field line. In this method, P(r, θ =0) is set to be $P_{sym}(r)$. We follow the field line from θ =0 and set P(r, θ =0) to the pressure along the field line. In either case, the two steps are iterated until the convergence of the island width is obtained.

4. Results

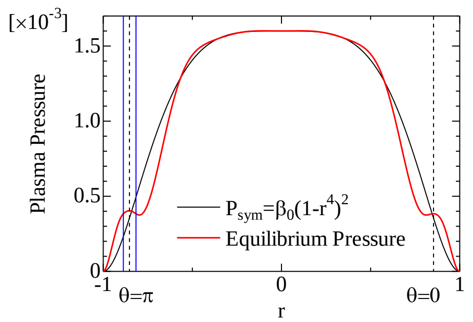
Figure1(a) shows the equilibrium pressure profile in the case with the diffusion equation. The initial pressure profile is $P_{sym} = \beta_0(1-r^4)^2$, where β_0 denotes the beta value at r=0. Here, β_0 = 0.16% is employed. Figure1(b) shows the equilibrium pressure profile in the case with the tracing field line. In both case, O-point of the island is located at $\theta = \pi$ and X-point is located at θ =0. Blue lines indicate the positions of separatrix of the magnetic island. Dotted lines are the position of the rational surface. In Fig.1(a), the pressure profile is locally flat at not only the O-point but also the X-point. In Fig.(b), the pressure profile is locally flat only at the O-point.

5. Summary

MHD equilibria including static magnetic islands for the reduced MHD equations are studied in straight heliotron plasmas. The equilibria with two kinds of pressure profile are found. One has local flat structure in the region corresponding to the island. In this case, the pressure gradient is kept at the X-point. The other has local flat structure that is almost uniform is the θ direction. In this case, the profile is locally flat even at the X-point.

The stability of the interchange modes for these equilibria is studied in a future.

(a)



(b)

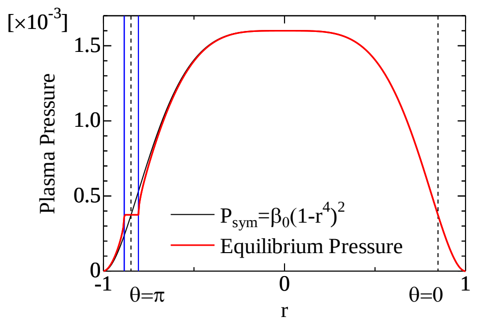


Figure1: Equibrium pressure profiles in the cases with methods of (a) diffusion equation and (b) tracing field line. Blue lines denote the position of the separatrix of the magnetic island. The dotted lines show the positions of the rational surface.

Acknowledgments

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