Dissipative Structures in Isotropic Hall MHD Turbulence

一様Hall MHD乱流における散逸構造

<u>Hideaki Miura</u>¹⁾, Keisuke Araki²⁾ 三浦 英昭¹⁾, 荒木 圭典²⁾

1)National Institute for Fusion Science, 322-6 Orsohi, Toki, Gifu 509-5292, Japan 1)核融合科学研究所 〒509-5292 岐阜県土岐市下石町322-6

2)Okayama University of Science, 1-1 Ridaicho, Okayama 700-0005, Japan 2)岡山理科大学 〒700-0005 岡山県岡山市理大町1-1

Dissipative structures in homogeneous, isotropic and incompressible Hall MHD turbulence are studied by means of direct numerical simulations. A data analysis by the use of the low-pass filter shows that the enstrophy density, which is the dissipative structure in the velocity field, is significantly modified at the moderate scales by the Hall effect. The large scale structures can be different between Hall and single-fluid MHD turbulence, while the small scale structures are similar to each other.

1. Introduction

Recent Magnetohydrodynamic (MHD) simulations resolve very fine scales numerically. However, the applicability of the single-fluid MHD equations is intrinsically restricted to some limited length and time scales in the sense of so-called the MHD ordering. For example, the length scales of the short wave number ballooning modes resolved in full-3D MHD simulations of the Large Helical Device can be shorter than the ion skin depth[1], and the description of such short wave number modes should be incomplete. Furthermore, the plasma motions in the small scales are somewhat turbulent in nonlinear time evolutions, and a physical model is required to describe small-scale turbulence more precisely. For such studies, we can consider studying by the use of either extended MHD or two-fluid equations.

The Hall-MHD model is among the simplest fluid models. The difference from the single-fluid MHD model appears only in the induction equation but it can bring about significant modifications of dynamics. For example, unstable Rayleigh-Taylor modes can be stabilized over some wave numbers [2]. The introduction of the Hall term also brings about the whistler-Alfven waves[3], which limit the time step width significantly in simulations. For the purpose of avoiding the time step width limitation, a hyper viscosity or a sort of numerical smoothing is often adopted. However, such an artificial truncation of small scales often changes plasma motions to those far from a well-resolved simulation. To avoid such a violent truncation, we need to clarify how the Hall term influences turbulent field and develop a smart numerical model. In this presentation, we study small scale structures in Hall MHD turbulence through direct comparison with single-fluid MHD turbulence.

2. Structure Analysis

Outlines of our numerical simulations are seen in Ref.[5]. The point of the analysis here is to make use of a low-pass filter and see spatial structures at each resolution. While a similar analysis has been carried out in Ref.[4] only for one cut-off wave number, we carry out here the analysis for various cut-off wave numbers and see how the hierarchy of turbulent structures appears or disappears. Here we focus on the dissipative structures, the enstrophy density $Q = |\mathbf{\omega}|^2 / 2$ and the current density $J = |\mathbf{j}|^2 / 2$ as the representatives of local structures where $\mathbf{\omega} = \nabla \times \mathbf{u}$ and $\mathbf{j} = \nabla \times \mathbf{B}$ are the vorticity and the current density vector, respectively.

In Fig.1, we see isosurfaces of Q and J. A region of $256^2 \times 128$ grid points is shown out of the 512^3 simulation box. In Fig.1(a), the low-pass filter is not operated. The spatial structures are sheet-like, as we often expect in isotropic MHD turbulence. In Fig.1(b), however, some of vortex structures (isosurfaces in dark color) are tubular, while the current structures (isosurfaces in bright gray) remain sheet-like. The point here is that neither the vortex structures nor the current structures in single-fluid MHD turbulence are modified significantly by operating the low-pass filter. Only the vortex structures in Hall MHD turbulence is modified by the low-pass filtering. (More detailed observations with some cut-off wave numbers are reported in Ref.[6].) It is noteworthy here that, the current-sheets are often observed at almost the same locations independent to the cut-off wave number, while tubular vortices do not necessarily appear in the same locations.



Fig.1: Isosurfaces of the enstrophy density Q and the current density J in Hall MHD turbulence, (a) without and (b) with low-pass filtering of $k_{x} = 32$.

In Fig.2, the kinetic energy transfer function of Hall MHD and single-fluid MHD turbulence are shown. While we usually normalize the wave number and the transfer function by the Kolmogorov length and the energy dissipation rate in order to compare two different computations, the comparison without the normalization in Fig.2 can make sense in this case because the initial condition of the two simulations is completely the same, and the difference comes only from the Hall term. It is clear that the energy transfer in Hall MHD turbulence is smaller than that in single-fluid MHD turbulence. Furthermore, it appear that the profile of the transfer function is shifted in the lower wave number side due to the Hall effect, because the zero-crossing wave number of the function in Hall MHD turbulence is lower than that of single-fluid MHD turbulence. Though the transfer function does not depict what kind of structures to be formed in the real space, the change of the profile should be closely related with the change of the spatial structures observed in Fig.1. Note here that the energy transfer functions at different time snap shots collapse to one profile (Hall MHD and single-fluid MHD respectively) by an appropriate normalization [7]. Thus the observation here can be applicable to all the decaying process after a transient stage before t=1.0 in Fig.2.

3. Summary

Numerical data analyses by the use of the low-pass filters are carried out. The scale-hierarchy of the vortex structures appears to have been changed by the Hall term, while that of the current structures does not appear being changed. Though further numerical studies on the scale-hierarchy should be required in order to construct appropriate numerical models to simulate Hall MHD events quickly, the observations presented here provide some insights for the purpose.



Fig.2. The kinetic energy transfer function of Hall MHD and the single-fluid MHD turbulence.

Acknowledgments

This work is performed under the auspices of the NIFS Collaboration Research Program (NIFS10KNSS011, NIFS11KNTS005, NIFS11KNSS016) and partially supported by KAKENHI (Grant-in-Aid for Scientific Research(C)) 22540509, 23340182 and 23540583.

References

- [1] H.Miura and N.Nakajima: Nuclear Fusion **50** (2010) 054006.
- [2] J.D.Huba and D. Winske: Phys. Plasmas 6 (1998) 2305.
- [3] S.Ohsaki and S.Mahajan: Phys. Plasmas 11(2004) 898.
- [4] H. Miura and D. Hori: Plasma Fusion Res. SERIES 8 (2009) 73.
- [5] D. Hori and H. Miura: Plasma Fusion Res. 3 (2008) S1053.
- [6] H. Miura and K. Araki: Proc. 13th European Turbulence Conference (Warsaw, Poland, Sep. 12-15, 2011), to appear in J. Physics: Conference Series.
- [7] K. Araki and H. Miura: to appear in Plasma Fusion Res. (2011).