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Multi-Scale Model for Interaction of Zonal Flows and Turbulence
ゾーナルフローと乱流相互作用の二重スケールモデル

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A multi-scale simulation model for the zonal flow and turbulence interactions is derived by means of the scale separation in the field-line-label coordinates for non-axisymmetric configurations. The entopy balance relation is generalized to the new approach, of which limiting forms are also discussed.

1. Introduction

Zonal flows and turbulence interactions have attracted many researchers' attention because the turbulent transport is regulated by the zonal flows [1]. Zonal flows characterized by constant electrostatic potential on flux surfaces in a torus are generated by the nonlinear interactions of turbulent eddies (Reynolds stress), and in turn feedback to the turbulent transport regulation.

Study of the zonal flows and turbulence in the toroidal plasma has made a large progress in the last two decades, where the numerical simulations of the plasma turbulent transport based on the gyrokinetic theory have contributed to enrichment of our understandings.

In axi-symmetric configurations, such as tokamaks, the zonal flows with long scale-length in poloidal and toroidal directions are often dealt as m =0 and n=0 mode, while the micro-turbulence, such as the ion temperature gradient (ITG) mode, is supposed to have high n numbers. The toroidal scale separation has naturally been taken into account in the flux tube model or in the wedge torus model in gyrokinetic simulations of turbulent transport.

In non-axisymmetric systems, however, the scale-separation is not trivial, as the n=0 mode couples with the non-axisymmetric component of the confinement field with the field-line-label (α) dependence. Also, the coupling of zonal flows and the non-axisymmetry is an origin of the zonal flow enhancement in case with the equilibrium-scale radial electric field (that is, a macro-scale poloidal ***E***x***B*** flow) [2-4]. A multi-scale approach to this issue is discussed in this article with initial results of gyrokinetic simulations applied to the zonal flow response enhancement in the model configuration of the Large Helical Device (LHD) [5] with the equilibrium-scale poloidal ***E***x***B*** flow.

2. Multi-Scale Model of Zonal flows and Turbulence

In the conventional flux tube model applied to non-axisymmetric configurations, the α dependence of the confinement field strength was not included, because of the locality assumption in the field-line-label direction [6, 7]. The flux tube model is justified by the weak α dependence of the linear dispersion relation of the ITG mode, and successfully applied to the turbulent transport in LHD, but with no equilibrium radial electric field E_r driving the poloidal ***E***x***B*** flow [6, 7].

In case with E_r, the zonal flow response function is enhanced with improvement of the collisionless orbits of helical-ripple-trapped particles [2, 3]. In the numerical simulation of the zonal flow enhancement, one needs to explicitly take account of the ***E***x***B*** advection of the non-zero toroidal mode number components coupled with the α dependence of the equilibrium field [4]. In contrast, the micro-turbulence has high n numbers with much smaller scale than the equilibrium scales. Thus, it is natural to introduce the scale-separation in the α direction between the zonal flow and turbulence components.

Taking average over the fast spatial scale (y) with the assumption of $\overline{\tilde{f}(x,y,z,v_{\|},\mu;\alpha')}=0$ and $\overline{\tilde{\Phi}(x,y,z,v_{\|};\alpha')}=0$ (where x, z, and α' denote the radial, parallel, and slow field-line-label coordinates, respectively), one finds the gyrokinetic equation for the zonal flow components of the perturbed ion gyrocenter distribution, $\hat{f}$,

$$\left[\frac{\partial}{\partial t}+v_{\|}\mathbf{b}\cdot\nabla+v_{dx}\frac{\partial}{\partial x}-\frac{\mu}{m}(\mathbf{b}\cdot\nabla B)\frac{\partial}{\partial v_{\|}}-\omega_{\theta}q\frac{\partial}{\partial \alpha'}\right]\hat{f} = \left(-v_{dx}\frac{\partial\hat{\Phi}}{\partial x}-v_{\|}\mathbf{b}\cdot\nabla\hat{\Phi}\right)\frac{e}{T}F_M+C(\hat{h})+S^{ZF} \quad . \quad (1)$$

Here, we assumed $\left|\partial\hat{f}/\partial x\right| >> \left|\partial\hat{f}/\partial\alpha'\right|$. The drift velocity v_{dx} and the magnetic field strength B include the field-line-label dependence. The poloidal rotation and the zonal flow drive by turbulence are, respectively, represented by ω_θ and S^{ZF}. Subtracting Eq. (1) from the original gyrokinetic equation for the perturbed ion gyrocenter distribution, the equation for the turbulence components $\tilde{f}$ is derived,

$$\left[\frac{\partial}{\partial t}+v_{\|}\mathbf{b}\cdot\nabla+\mathbf{v}_d\cdot\nabla-\frac{\mu}{m}(\mathbf{b}\cdot\nabla B)\frac{\partial}{\partial v_{\|}}-\omega_\theta r_0\frac{\partial}{\partial y}\right]\tilde{f} \tag{2}$$

$$+\frac{c}{B_0}\left\{\hat{\Phi}+\tilde{\Phi},\,\hat{f}+\tilde{f}\right\}=\left(\mathbf{v}_*-\mathbf{v}_d-v_{\|}\mathbf{b}\right)\cdot\frac{e\nabla\Phi}{T}F_M+C(h)-S^{ZF}$$

where the nonlinear term is denoted by the Poisson brackets. The source term for the zonal flows is defined as $S^{ZF}=-\frac{c}{B_0}\overline{\left\{\hat{\Phi}+\tilde{\Phi},\,\hat{f}+\tilde{f}\right\}}$ which describes interactions of zonal flows and turbulence.

A discrete form of Eq. (1) for the field-line-label coordinates leads to a "flux tube bundle" model for zonal flows and turbulence in the non-axisymmetric system, where Eq. (2) is solved for each ith flux tube corresponding to the discretized α'_i coordinate. Figure 1 depicts a schematic view of the flux tube bundle model consisting of multiple flux tubes and zonal flows with the α' coordinate dependence.

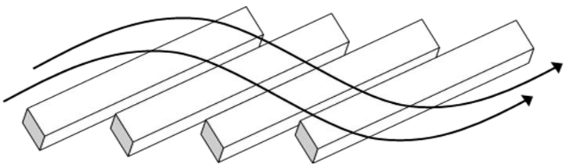

Fig 1: A schematic view of the flux tube bundle model. Flux tube simulation boxes, in which Eq.(2) is solved, are set at $\alpha'=\alpha'_i$, where the zonal flow drive (S^{ZF}_i) is calculated in each flux tube.

3. Limiting Cases

For the axisymmetric configuration, the model equations, Eqs. (1) and (2) reduce to the standard gyrokinetic equation for the perturbed ion gyrocenter distribution. Thus, the present model is a natural extension of the conventional flux tube model for the non-axisymmetric systems.

In case with no equilibrium radial electric field, E_r =0, we also recover the conventional flux tube model for the non-axisymmetric case, if a single flux tube is considered.

The zonal flow response enhancement by E_r is reproduced by solving the linearized version of Eq. (1) with S^{ZF} = 0. Actually, we have confirmed the amplification of the residual zonal flow in agreement with the results given in Ref. [4].

4. Entropy Balance

The entropy balance relation of the flux tube bundle and the zonal flows can also be derived. For the ith flux tube, one finds

$$\frac{d}{dt}\left(\delta\tilde{S}_i+\tilde{W}_i\right)=L_T^{-1}Q_i+\tilde{D}_i-\tau^{ZF} \tag{3}$$

where τ^{ZF} denotes the entropy transfer from turbulence to zonal flows [8]. The entropy balance for the zonal flows at $\alpha'=\alpha'_i$ is given by

$$\frac{\partial}{\partial t}\left(\delta\hat{S}_i+\hat{W}_i\right)-\omega_\theta q\frac{\partial}{\partial\alpha'}\left(\delta\hat{S}_i+\hat{W}_i\right)=\hat{D}_i+\tau^{ZF}\quad, \tag{4}$$

where the entropy advection term due to the poloidal rotation appears in the left hand side. Summation over i for turbulence and zonal flows leads to the total entropy balance relation, that is,

$$\frac{d}{dt}(\delta S+W)=L_T^{-1}Q+D\quad, \tag{5}$$

with the same form as that in the conventional flux tube model.

5. Summary

A multi-scale simulation model for zonal flow and turbulence interactions in non-axisymmetric systems has been developed. The scale separation in the field-line-label coordinates leads to a coupled set of gyrokinetic equations. The new approach describes a natural extension of the flux tube model.

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