# 23P134-B

# Higher Order Terms of the Guiding-Center Transformation and the Gyrokinetic Quasi-Neutrality Condition
# 案内中心変換の高次項とジャイロ運動論的準中性条件


Naoaki Miyato[1], Bruce D. Scott[2] and Masatoshi Yagi[1,3]
宮戸直亮[1], スコット・ブルース[2], 矢木雅敏[1,3]

[1]*Japan Atomic Energy Agency*
*2-116 Omotedate, Obuchi, Rokkasho, Kamikita-gun, Aomori 039-3212, Japan*
原子力機構 〒039-3212 青森県上北郡六ヶ所村尾駮表舘2-166
[2]*Max-Planck-Institut für Plasmaphysik*
*D-85748 Garching, Germany*
マックス・プランクプラズマ物理研究所 ドイツ連邦共和国バイエルン州ガルヒンクD-85748
[3]*Kyushu University*
*6-1 Kasuga, Fukuoka 816-8580 Japan*
九州大学 〒816-8580 福岡県春日市春日公園6-1



The standard gyrokinetic model, which is originally formulated for perturbations with short wavelength and small amplitude, is not always valid in the long wavelength regime. The reduced (gyrokinetic) Poisson equation or the gyrokinetic quasi-neutrality condition in the standard model is no longer sufficient to obtain electrostatic potential in the long wavelength regime. Since the polarization term including the electrostatic potential goes to higher order, the other higher order terms which are not considered in the standard model are needed to obtain the electrostatic potential. Taking into account a higher order displacement vector associated with the guiding-center transformation, we find additional higher order terms coming from nonuniformity of magnetic field in the gyrokinetic Poisson equation and the quasi-neutrality condition.


## 1. Introduction

Control of anomalous transport is one of very important issues in magnetized fusion plasma research. For this purpose, it is indispensable to clarify mechanism of formation of transport barriers [1]. It is believed that drift wave type microturbulence driven by density and/or temperature gradient is a principal cause of the anomalous transport [2]. Since spatiotemporal scale of the drift wave turbulence is small compared to that of background profiles including mean flow, it leads to separate treatment of the drift wave turbulence and evolution of the background profiles. However, global simulation handling both the turbulence and the profile evolution is necessary for ultimate understanding of the formation of transport barriers. The standard gyrokinetic model is formulated for investigation of the drift wave turbulence, and perturbations with short wavelength ($k_\perp \rho \sim 1$) and small amplitude ($e\varphi/T \ll 1$) (gyrokinetic ordering) are assumed in the formulation of the gyrokinetic model [3-5]. Therefore, there is no guarantee that the standard gyrokinetic model is valid in the long wavelength regime as well. Although it was claimed that the standard gyrokinetic model would be also valid in the long wavelength by reinterpretation of the gyrokinetic ordering [6], it was reported later that the standard gyrokinetic model is not necessarily sufficient in the long wavelength regime [7]. We investigate this issue from a point of view of push-forward representation associated with phase space transformation [8,9] and show that additional terms stemming from nonuniformity of magnetic field would be important in gyrokinetic quasi-neutrality condition in the long wavelength regime.

## 2. Gyrokinetic model

Modern formulation of the gyrokinetic model is based on the phase space Lagrangian Lie-transform perturbation method [10] and consists of two-step phase space transformation from the particle phase space to the gyro-center phase space [4-5]. In the first step the gyro-phase dependence of a gyrating particle in equilibrium magnetic field is removed by the phase space transformation from the particle phase space to the guiding-center phase space (guiding-center transformation) [11]. Smallness parameter of the guiding-center transformation is $\epsilon \sim \rho/L \ll 1$ where $\rho$ is the Larmor radius of a particle and $L$ is the scale length of the magnetic field. After the first step time-dependent electromagnetic perturbations are introduced into the system and the gyro-phase angle dependence reintroduced with the perturbations is removed by the transformation from the guiding-center phase space to the gyro-center phase space (gyro-center transformation). Here smallness parameter is $\epsilon_\delta \sim e\varphi/T \ll 1$. As mentioned in Introduction, the standard gyrokinetic model is originally constructed under the gyrokinetic ordering ( $k_\perp \rho \sim 1$ , $e\varphi/T \ll 1$). This condition can be interpreted as $k_\perp \rho e\varphi/T \ll 1$ which means that the E×B drift velocity is much slower than the thermal velocity. The slow flow condition is also satisfied in the long wavelength regime ($k_\perp\rho \ll 1$, $e\varphi/T \sim 1$). This reinterpretation of the ordering seems to be no problem in the phase space transformation for the single particle dynamics and in the gyrokinetic Vlasov equation. The change of the ordering, however, has an effect on the polarization term including $\varphi$ in the gyrokinetic quasi-neutrality condition (and the gyrokinetic Poisson equation)

$$n_e = \bar{N}_i + n_{i0}(\Gamma_0 - 1)\frac{e\varphi}{T_i}.$$

In the equation the ion particle density is expressed in terms of the gyro-center variables and it is called the push-forward representation of particle density. The representation depends on details of the guiding-center and gyro-center transformations:

$$\mathbf{x} = \bar{\mathbf{X}} + \epsilon\bar{\boldsymbol{\rho}} + \epsilon_\delta\epsilon\bar{\boldsymbol{\rho}}_{\mathrm{gy}} + \epsilon^2\bar{\boldsymbol{\rho}}_{\mathrm{B}}\cdots.$$

The polarization term comes from the gyro-center displacement vector $\bar{\boldsymbol{\rho}}_{\mathrm{gy}}$ associated with the *gyro-center transformation* at $O(\epsilon\epsilon_\delta)$. In the long wavelength regime the polarization term goes to higher order than the standard gyrokinetic case. Hence, we have to take into account the other higher order terms to obtain long wavelength component of the electrostatic potential. Then, we have to consider another displacement vector associated with the *guiding-center transformation* $\bar{\boldsymbol{\rho}}_{\mathrm{B}}$ as well [9, 12]. This piece is not considered in the standard model.

## 3. Higher order displacement vector

The guiding-center transformation of particle position $\mathbf{x}$ is written in general as

$$\mathbf{x} = \mathbf{X} - \epsilon G_1^{\mathbf{X}} - \epsilon^2\left(G_2^{\mathbf{X}} - \frac{1}{2}\mathbf{G}_1\cdot\mathbf{d}G_1^{\mathbf{X}}\right) + \cdots,$$

where $\mathbf{G}_n$ is the nth-order vector field generating the guiding-center transformation and $\mathbf{G}_n\cdot\mathbf{d} = G_n^j\partial_j$. Negative of $G_1^{\mathbf{X}}$ is the usual gyroradius vector. We denote the second order piece as

$$\boldsymbol{\rho}_B = -\left(G_2^{\mathbf{X}} - \frac{1}{2}\mathbf{G}_1\cdot\mathbf{d}G_1^{\mathbf{X}}\right).$$

Since explicit forms of $\mathbf{G}_n$ and $\boldsymbol{\rho}_B$ are very complicated [11, 13], they are not shown here. Considering $\boldsymbol{\rho}_B$, we obtain additional terms in the push-forward representation of particle density, therefore, in the gyrokinetic quasi-neutrality condition:

$$-\int d^6\bar{\mathbf{Z}}\delta^3(\bar{\mathbf{X}} - \mathbf{r})\nabla\cdot J(\bar{\mathbf{Z}})\bar{F}\langle\bar{\boldsymbol{\rho}}_B\rangle,$$

where $\langle\cdot\rangle$ denotes gyro-phase average. The gyro-phase angle average of $\boldsymbol{\rho}_B$ is important for the quasi-neutrality condition and is given by

$$\langle\bar{\boldsymbol{\rho}}_B\rangle = -\left[\frac{\mu B}{m\Omega^2}\frac{1}{2}(\nabla\cdot\hat{b})\hat{b} + \frac{U^2}{\Omega^2}\hat{b}\cdot\nabla\hat{b} + \frac{3}{2}\frac{\mu B}{m\Omega^2}\nabla_\perp \log B\right].$$

A similar result is found in Ref. [12] in which the first term is missing. Although the first term can be neglected in some cases, it may be important in tokamaks with relatively large ripples of the toroidal magnetic field. The second term is common to the result in Ref. [12], while the coefficient in front of the last term is not. Effects of the additional terms on electrostatic potential will be investigated in future.


## Acknowledgments

This work is partly supported by Grant-in-Aid for Young Scientist (B) from Japan Society for the Promotion of Science (22760663).


## References


[1] J. W. Connor, T. Fukuda, X. Garbet, C. Gormezano, V. Mukhovatov, M. Wakatani, the ITB Database Group and the ITPA Topical Group on Transport and Internal Barrier Physics: Nucl. Fusion **44** (2004) R1.
[2] W. Horton: Rev. Mod. Phys. **71** (1999) 735.
[3] D. H. E. Dubin, J. A. Krommes, C. Oberman and W. W. Lee: Phys Fluids **26** (1983) 3524.
[4] T. S. Hahm: Phys. Fluids **31** (1988) 2670.
[5] A. Brizard: J. Plasma Phys. **41** (1989) 541.
[6] A. M. Dimits, L. L. LoDestro and D. H. E. Dubin: Phys. Fluids B **4** (1992) 274.
[7] F. I. Parra and P. J. Catto: Plasma Phys. Control. Fusion **50** (2008) 065014.
[8] N. Miyato, B. D. Scott and S. Tokuda: J. Plasma Fusion Res. SERIES **9** (2010) 546.
[9] N. Miyato and B. D. Scott: "Fluid moments in the reduced model for plasmas with large flow velocity," submitted to Plasma Fusion Res.
[10] J. R. Cary and R. G. Littlejohn: Ann. Phys. (N.Y.) **151** (1983) 1.
[11] R. G. Littlejohn: Phys. Fluids **24** (1981) 1730.
[12] A. J. Brizard and N. Tronko: Phys. Plasmas **18** (2011) 082307.
[13] A. J. Brizard and A. Mishchenko: J. Plasma Phys. **75** (2009) 675.