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Analysis of plasma rotation effects on ballooning stability in magnetospheric plasma confinement
磁気圏型プラズマ閉じ込めのバルーニング安定性に対する流れの効果の分析

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Magnetohydrodynamics ballooning mode stability of magnetospheric plasma configuration is studied via time-dependent eikonal formulation. Plasma rotation effects on the ballooning mode stability can be categorized into four: (i) change of equilibrium which enters the ballooning equation through the metric elements, (ii) new self-adjoint terms in the ballooning equation originating from plasma rotation, (iii) time varying wave number due to rotation shear and (iv) non-self-adjoint terms including first-order time derivative. (i) and (ii) modifies the instability growth rate in the order square of Mach number, rotation speed divided by thermal speed. (iii) leads to stabilization after long time similar to slab geometry, not in a sense of time average as tokamak case. (iv) partly works like friction.

1. Introduction

Plasma confinement by magnetospheric configuration was proposed to achieve advanced fuel nuclear fusion[1], and has been studied in laboratories toward the goal[2,3]. Since these experimental devices have only poloidal magnetic field, pressure-driven magnetohydrodynamics (MHD) instabilities might occur. Thus the MHD stability was studied for limiter [4] and separatrix configurations[5]. The separatrix was shown to have stabilizing effect because of its big flux expansion[5]. These studies assumed static plasma equilibria. If a plasma is rotating, the pressure-driven instability is affected considerably. For example, rotation shear stabilizes ballooning mode[6,7] in tokamaks on time average[8-11]. The stabilization occurs due to energy transfer from unstable to stable modes[11].

These studies mainly focused on plasma rotation shear. However, the magnitude of the rotation can also play a role. In the present paper, we study how to categorize the plasma rotation effects on ballooning instabilities by examining the governing equation. We also point out an important difference between the magnetospheric configuration and tokamaks. The conclusions given below have been verified by numerical simulations, which will be presented elsewhere.

2. Examination of governing equation

We consider a magnetospheric, axisymmetric plasma with only poloidal magnetic field and toroidal rotation. The equilibrium is described by the Grad–Shafranov equation including the toroidal rotation[12],

$$R^2\nabla\cdot\left(\frac{1}{R^2}\nabla\psi\right) = -\frac{\beta_0}{2}R^2\left.\frac{\partial p}{\partial\psi}\right|_R. \qquad (1)$$

Note that all quantities are normalized by their typical values in this paper. The equilibrium magnetic field is expressed by $\mathbf{B} = \nabla\psi\times\nabla\phi$, where ϕ is the toroidal angle. The equilibrium toroidal rotation speed is given by $R\Omega(\psi)$. The ratio of plasma pressure to magnetic pressure is denoted by $\beta_0 := 2\mu_0 p_0/B_0^2$, where p_0 and B_0 are typical values of pressure and magnetic field, respectively. The pressure takes the form $p = \bar{p}(\psi)\exp[M_0^2\Omega^2(\psi)(R^2/R_\mathrm{c}^2 - 1)]$ when the temperature T is constant on each magnetic surface. Here $M_0 := R_\mathrm{c}\Omega_0/\sqrt{2T(\psi)/m_\mathrm{i}}$ is the Mach number based on thermal velocity of ions. The ion mass is m_i, R_c is the major radius of the internal ring current, and Ω_0 is the typical value of toroidal rotation frequency. The plasma rotation changes the source term of Eq. (1) and thus the solution ψ too. The change of the equilibrium, or the metric elements equivalently, enters the ballooning equation explained below through the change of its coefficients. Assuming $M_0^2 \ll 1$, we may expand the source term and obtain

$$\left.\frac{\partial p}{\partial\psi}\right|_R = \bar{p}' + \mathcal{O}(M_0^2), \qquad (2)$$

where the prime denotes ψ derivative. Since the source term changes in $\mathcal{O}(M_0^2)$, the metric elements also change in $\mathcal{O}(M_0^2)$.

Next we examine the linearized ideal MHD equation including equilibrium plasma flow[13]:

$$\rho \frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} + 2M_{\mathrm{A}} \rho \mathbf{v} \cdot \nabla \frac{\partial \boldsymbol{\xi}}{\partial t} = \mathcal{F}_{\mathrm{f}}(\boldsymbol{\xi}), \tag{3}$$

$$\begin{aligned} \mathcal{F}_{\mathrm{f}} := & (\nabla \times \tilde{\mathbf{B}}) \times \mathbf{B} + (\nabla \times \mathbf{B}) \times \tilde{\mathbf{B}} \\ & + \frac{\beta_0}{2} \nabla (\boldsymbol{\xi} \cdot \nabla p + \Gamma p \nabla \cdot \boldsymbol{\xi}) \\ & + M_{\mathrm{A}}^2 \nabla \cdot (\rho \boldsymbol{\xi} \mathbf{v} \cdot \nabla \mathbf{v} - \rho \mathbf{v} \mathbf{v} \cdot \nabla \boldsymbol{\xi}), \end{aligned} \tag{4}$$

where ρ and $\mathbf{v}$ are equilibrium mass density and flow velocity, respectively. The specific heat ratio is Γ. A displacement of the plasma element from the equilibrium trajectory is $\boldsymbol{\xi}$, defined via a perturbed flow velocity $\tilde{\mathbf{v}} =: \partial \boldsymbol{\xi}/\partial t + \mathbf{v} \cdot \nabla \boldsymbol{\xi} + \boldsymbol{\xi} \cdot \nabla \mathbf{v}$. A perturbed magnetic field is given by $\tilde{\mathbf{B}} := \nabla \times (\boldsymbol{\xi} \times \mathbf{B})$. The Alfvén Mach number is defined by $M_{\mathrm{A}} := \Omega_0 \tau_{\mathrm{A}}$ with $\tau_{\mathrm{A}} := L/(B_0/\sqrt{\mu_0 \rho})$ and L a typical length. While the second term of the l.h.s. of Eq. (3) introduces anti-self-adjointness, the r.h.s. of Eq. (3), a generalization of the MHD force operator[14], is still self-adjoint even including the new terms of rotation $\mathbf{v}$. The ratio of the rotation term to the pressure term is roughly M_0^2 by the relation $M_{\mathrm{A}}^2 = \beta_0 M_0^2$. Thus we expect an $\mathcal{O}(M_0^2)$ modification of growth rate for pressure-driven instabilities by the new terms, which is the same order as the equilibrium change.

For ballooning modes, we adopt the eikonal formulation based on a large toroidal mode number n[6,7]. Using $n \gg 1$, we express $\boldsymbol{\xi} = \sum_j \left(n^{-j} \hat{\boldsymbol{\xi}}^{(j)} \right) \mathrm{e}^{\mathrm{i}\, nS}$, where $\hat{\boldsymbol{\xi}}^{(j)}$ represents an envelope and S is an eikonal. We assume $\mathbf{B} \cdot \nabla S = 0$ and $\partial S/\partial t + \mathbf{v} \cdot \nabla S = 0$ for the eikonal[8-11], to obtain $S = -\phi + M_{\mathrm{A}} \Omega(\psi) t + S_0(\psi)$. The wave vector then becomes $\hat{\mathbf{k}} := \nabla S = -\nabla \phi + \tilde{k}_\psi \nabla \psi$, and the radial wave number is given by

$$\tilde{k}_\psi = M_{\mathrm{A}} \Omega' t + k_{\psi 0}, \qquad k_{\psi 0} := S_0'. \tag{5}$$

The t dependence of $\tilde{k}_\psi$ expresses the stretch of wave in time by rotation shear. The rotation shear determines the time scale of $\tilde{k}_\psi$ variation as $(M_{\mathrm{A}} \Omega')^{-1}$. Note that this $\hat{\mathbf{k}}$ does not have dynamical lattice symmetry[9] as in tokamak case, leading to similar behavior as in slab geometry[15]. Collecting terms at each order in n, we obtain $\hat{\boldsymbol{\xi}}^{(0)} = \xi_\parallel \mathbf{B} + \xi_\perp \mathbf{B} \times \hat{\mathbf{k}}/B^2$ and the coupled wave equations for $x = (\xi_\parallel, \xi_\perp)^{\mathrm{T}}$, called the ballooning equation. Although the explicit representation of the equation will be presented elsewhere, the abstract form is as follows:

$$\mathcal{M}_2 \frac{\partial^2 x}{\partial t^2} + \mathcal{M}_1 \frac{\partial x}{\partial t} + \mathcal{M}_0 x = \mathcal{L} x, \tag{6}$$

where $\mathcal{M}_j$s and $\mathcal{L}$ are matrices of operators. The l.h.s. (r.h.s.) of Eq. (6) comes from the l.h.s. (r.h.s.) of Eq. (3). The $\mathcal{M}_1$ term yields an effect analogous to friction[16]. The operator $\mathcal{L}$ has $\mathcal{O}(M_0^2)$ terms as similar to $\mathcal{F}_{\mathrm{f}}$. Most important point is that these operators include t through $\hat{\mathbf{k}}$; Eq. (6) is non-autonomous. However, if we consider t just as a parameter, we find that $\mathcal{L}$ is still self-adjoint. Thus we may be able to utilize the spectral decomposition of $\mathcal{L}$ at each instance to analyze the solution of Eq. (6). If the time scale of the dynamics is much faster than that of $\tilde{k}_\psi$ variation, the wave evolves as an eigenmode of $\mathcal{L}$ at each instance.

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